Arithmetic Progression

Question 1.

Which of the following sequences are in arithmetic progression?

(i) 2, 6, 10, 14,

(ii) 15, 12, 9, 6,

(iii) 5, 9, 12, 18,

(iv) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\ldots$

Solution:

(i) 2, 6, 10, 14, ...

$d_1 = 6 - 2 = 4$

$d_2 = 10 - 6 = 4$

$d_3 = 14 - 10 = 4$

Since $d_1 = d_2 = d_3$, the given sequence is in arithmetic progression.

(ii) 15, 12, 9, 6, ...

$d_1 = 12 - 15 = -3$

$d_2 = 9 - 12 = -3$

$d_3 = 6 - 9 = -3$

Since $d_1 = d_2 = d_3$, the given sequence is in arithmetic progression.

(iii) 5, 9, 12, 18, ...

$d_1 = 9 - 5 = 4$

$d_2 = 12 - 9 = 3$

Since $d_1 \neq d_2$, the given sequence is not in arithmetic progression.

(iv) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\ldots$

$d_1 = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = -\frac{1}{6}$

$d_2 = \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = -\frac{1}{12}$

Since $d_1 \neq d_2$, the given sequence is not in arithmetic progression.

Question 2.
The nth term of sequence is (2n – 3), find its fifteenth term.

Solution:

n^{th} term of A.P. = (2n – 3)

$\Rightarrow t_n = 2n - 3$

$\therefore 15^{th}$ term = $t_{15} = 2 \times 15 - 3 = 30 - 3 = 27$

Question 3.
If the pth term of an A.P. is (2p + 3), find the A.P.

Solution:

p^{th} term of an A.P. = 2p + 3

$\Rightarrow t_p = 2p + 3$

Putting t = 1, 2, 3, ..., we get

$t_1 = 2 \times 1 + 3 = 2 + 3 = 5$

$t_2 = 2 \times 2 + 3 = 4 + 3 = 7$

$t_3 = 2 \times 3 + 3 = 6 + 3 = 9$ and so on.

Thus, the A.P. is 5, 7, 9, ...

Question 4.
Find the 24th term of the sequence:
12, 10, 8, 6,……

Solution:

The given sequence is 12, 10, 8, 6,....

Now,

$10 - 12 = -2$

$8 - 10 = -2$

$6 - 8 = -2$, etc.

Hence, the given sequence is an A.P. with first term a = 12 and common difference d = −2.

The general term of an A.P. is given by

$t_n = a + (n - 1)d$

$\Rightarrow t_{24} = 12 + (24 - 1)(-2) = 12 + 23 \times (-2) = 12 - 46 = -34$

So, the 24^{th} term is −34.

Question 5.

Find the 30th term of the sequence:

$\frac{1}{2}, 1, \frac{3}{2}, \ldots\ldots\ldots$

Solution:

The given sequence is $\frac{1}{2}, 1, \frac{3}{2}, \ldots\ldots$

Now,

$1 - \frac{1}{2} = \frac{1}{2}$

$\frac{3}{2} - 1 = \frac{1}{2}$, etc.

Hence, the given sequence is an A.P. with first term $a = \frac{1}{2}$

and common difference $d = \frac{1}{2}$.

The general term of an A.P. is given by

$t_n = a + (n-1)d$

$\Rightarrow t_{30} = \frac{1}{2} + (30-1)\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{29}{2} = \frac{30}{2} = 15$

So, the 30th term is 15.

Question 6.

Find the 100th term of the sequence

$\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots\ldots\ldots$

Solution:

The given A.P. is $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots\ldots$

Now,

$2\sqrt{3} - \sqrt{3} = \sqrt{3}$

$3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$, etc.

Hence, the given sequence is an A.P. with first term $a = \sqrt{3}$

and common difference $d = \sqrt{3}$.

The general term of an A.P. is given by

$t_n = a + (n-1)d$

$\Rightarrow t_{100} = \sqrt{3} + (100-1) \times \sqrt{3} = \sqrt{3} + 99\sqrt{3} = 100\sqrt{3}$

So, the 100th term is $100\sqrt{3}$.

Question 7.

Find the 50th term of the sequence:

$\frac{1}{n}, \frac{n+1}{n}, \frac{2n+1}{n}, \ldots\ldots\ldots$

Solution:

The given sequence is $\frac{1}{n}, \frac{n+1}{n}, \frac{2n+1}{n}, \ldots.$

Now,

$\frac{n+1}{n} - \frac{1}{n} = \frac{n+1-1}{n} = \frac{n}{n} = 1$

$\frac{2n+1}{n} - \frac{n+1}{n} = \frac{2n+1-n-1}{n} = \frac{n}{n} = 1$, etc.

Hence, the given sequence is an A.P. with first term $a = \frac{1}{n}$ and common difference d = 1.

The general term of an A.P. is given by

$t_n = a + (n-1)d$

$\Rightarrow t_{50} = \frac{1}{n} + (50-1)(1) = \frac{1}{n} + 49$

So, the 50th term is $\frac{1}{n} + 49$.

Question 8.

Is 402 a term of the sequence :

8, 13, 18, 23,…………?

Solution:

The given sequence is 8, 13, 18, 23, ….

Now,

13 – 8 = 5

18 – 13 = 5

23 – 18 = 5, etc.

Hence, the given sequence is an A.P. with first term a = 8 and common difference d = 5.

The general term of an A.P. is given by

$t_n = a + (n - 1)d$

$\Rightarrow 402 = 8 + (n - 1)(5)$

$\Rightarrow 394 = 5n - 5$

$\Rightarrow 399 = 5n$

$\Rightarrow n = \frac{399}{5}$

The number of terms cannot be a fraction.

So clearly, 402 is not a term of the given sequence.

Question 9.

Find the common difference and 99^{th} term of the arthimetic progression :

$7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}, \ldots\ldots\ldots\ldots$

Solution:

Find the common difference and 99^{th} term of the arthimetic progression :

The given A.P. is $7\frac{3}{4},\ 9\frac{1}{2},\ 11\frac{1}{4}, \ldots..$

i.e. $\frac{31}{4}, \frac{19}{2}, \frac{45}{4}, \ldots\ldots$

Common difference $= d = \frac{19}{2} - \frac{31}{4} = \frac{38 - 31}{4} = \frac{7}{4} = 1\frac{3}{4}$

First term $= a = \frac{31}{4}$

The general term of an A.P. is given by

$t_n = a + (n - 1)d$

$\Rightarrow t_{99} = \frac{31}{4} + (99 - 1) \times \frac{7}{4} = \frac{31}{4} + 98 \times \frac{7}{4} = \frac{31}{4} + \frac{686}{4} = \frac{717}{4} = 179\frac{1}{4}$

Question 10.

How many terms are there in the series :

(i) 4, 7, 10, 13, …………, 148?

(ii) 0.5, 0.53, 0.56, ……………, 1.1?

(iii) $\frac{3}{4}, 1, 1\frac{1}{4}, \ldots\ldots\ldots\ldots, 3$

Solution:

(i) The given series is 4, 7, 10, 13, ..., 148

$7 - 4 = 3,\ 10 - 7 = 3,\ 13 - 10 = 3$, etc

Thus, the given series is an A.P. with first term a = 4 and common difference d = 3.

Last term = l = 148

$4 + (n-1)(3) = 148$

$\Rightarrow (n-1) \times 3 = 144$

$\Rightarrow n - 1 = 48$

$\Rightarrow n = 49$

Thus, there are 49 terms in the given series.

(ii) The given series is 0.5, 0.53, 0.56, ..., 1.1

$0.53 - 0.5 = 0.03,\ 0.56 - 0.53 = 0.03$, etc.

Thus, the given series is an A.P. with first term a = 0.5 and common difference d = 0.03

Last term = l = 1.1

$0.5 + (n-1)(0.03) = 1.1$

$\Rightarrow (n-1) \times 0.03 = 0.6$

$\Rightarrow n - 1 = 20$

$\Rightarrow n = 21$

Thus, there are 21 terms in the given series.

(iii) The given seres is $\frac{3}{4}, 1, 1\frac{1}{4}, .., 3 \Rightarrow \frac{3}{4}, 1, \frac{5}{4}, ..., 3$

$1 - \frac{3}{4} = \frac{1}{4},\ \frac{5}{4} - 1 = \frac{1}{4}$, etc

Thus, the given series is an A.P. with first term $a = \frac{3}{4}$

and common difference $d = \frac{1}{4}$.

Last term = l = 3

$\frac{3}{4} + (n-1)\left(\frac{1}{4}\right) = 3$

$$\Rightarrow (n-1)\times\frac{1}{4} = 3-\frac{3}{4}$$
$$\Rightarrow (n-1)\times\frac{1}{4} = \frac{9}{4}$$
$$\Rightarrow n-1 = 9$$
$$\Rightarrow n = 10$$
Thus, there are 10 terms in the given series.

Question 11.
Which term of the A.P. 1 + 4 + 7 + 10 + ………. is 52?

Solution:

The given A.P. is $1+4+7+10+\ldots$

Here, first term $a = 1$ and common difference $d = 4-1 = 3$

Let n^{th} term of the given A.P. be 52.

$\Rightarrow 52 = a+(n-1)d$

$\Rightarrow 52 = 1+(n-1)\times 3$

$\Rightarrow 51 = (n-1)\times 3$

$\Rightarrow n-1 = 17$

$\Rightarrow n = 18$

Thus, the 18^{th} term of the given A.P. is 52.

Question 12.
If 5th and 6th terms of an A.P are respectively 6 and 5. Find the 11th term of the A.P
Solution:

The general term of an A.P. is given by

$t_n = a+(n-1)d$

Now, $t_5 = 6$

$\Rightarrow a+(5-1)d = 6$

$\Rightarrow a+4d = 6$ ....(i)

And, $t_6 = 5$

$\Rightarrow a+(6-1)d = 5$

$\Rightarrow a+5d = 5$ ....(ii)

Subtracting (ii) from (i), we get

$-d = 1$
$\Rightarrow d = -1$
Substituting $d = -1$ in (i), we get
$a + 4(-1) = 6$
$\Rightarrow a - 4 = 6$
$\Rightarrow a = 10$
$\Rightarrow t_n = 10 + (n - 1)(-1)$
$\Rightarrow t_{11} = 10 + (11 - 1)(-1) = 10 - 10 = 0$

Question 13.

If t_n represents n^{th} term of an A.P, $t_2 + t_5 - t_3 = 10$ and $t_2 + t_9 = 17$, find its first term and its common differenc

Solution:

Let the first term of an A.P. be a and the common difference be d.
The general term of an A.P. is given by $t_n = a + (n - 1)d$
Now, $t_2 + t_5 - t_3 = 10$
$\Rightarrow (a + d) + (a + 4d) - (a + 2d) = 10$
$\Rightarrow a + d + a + 4d - a - 2d = 10$
$\Rightarrow a + 3d = 10 \quad \ldots.(i)$
Also, $t_2 + t_9 = 17$
$\Rightarrow (a + d) + (a + 8d) = 17$
$\Rightarrow 2a + 9d = 17 \quad \ldots.(ii)$
Multiplying equation (i) by 2, we get
$2a + 6d = 20 \quad \ldots.(iii)$
Subtracting (ii) from (iii), we get
$-3d = 3$
$\Rightarrow d = -1$
Substituting value of d in (i), we get
$a + 3(-1) = 10$
$\Rightarrow a - 3 = 10$
$\Rightarrow a = 13$
Hence, $a = 13$ and $d = -1$.

Question 14.
Find the 10th term from the end of the A.P. 4, 9, 14,…….., 254

Solution:

The given A.P. is 4, 9, 14, ….., 254.
First term = 4
Common difference = 9 − 4 = 5
Last term = l = 254
For the reverse A.P., first term = 254 and common difference = −5
Thus, 10^{th} term from the end of an given A.P.
= 10^{th} term from the beginning of its reverse A.P.
$= 254 + (10 - 1) \times (-5)$
$= 254 - 45$
$= 209$

Question 15.
Determine the arithmetic progression whose 3rd term is 5 and 7th term is 9.

Solution:

For an A.P.,
$t_3 = 5$
$\Rightarrow a + 2d = 5 \quad ....(i)$
And, $t_7 = 9$
$\Rightarrow a + 6d = 9 \quad ....(ii)$
Subtracting (i) from (ii), we get
$4d = 4$
$\Rightarrow d = 1$
Substituting d = 1 in (i), we get
$a + 2 \times 1 = 5$
$\Rightarrow a = 3$
Thus, the required A.P. = a, a + d, a + 2d, a + 3d, …..
= 3, 4, 5, 6, …..

Question 16.
Find the 31st term of an A.P whose 10th term is 38 and 10th term is 74.

Solution:

The general term of an A.P. is given by
$t_n = a + (n-1)d$
Now, $t_{10} = 38$
$\Rightarrow a + 9d = 38 \quad ....(i)$
And, $t_{16} = 74$
$\Rightarrow a + 15d = 74 \quad ....(ii)$
Subtracting (i) from (ii), we get
$6d = 36$
$\Rightarrow d = 6$
Substituting d = 6 in (i), we get
$a + 9 \times 6 = 38$
$\Rightarrow a + 54 = 38$
$\Rightarrow a = -16$
$\Rightarrow t_n = -16 + (n-1)(6)$
$\Rightarrow t_{31} = -16 + 30 \times 6 = -16 + 180 = 164$

Question 17.
Which term of the services :
21, 18, 15, ………… is – 81?
Can any term of this series be zero? If yes find the number of term.

Solution:

The given A.P. is 21, 18, 15, ...
Here, first term a = 21 and common difference d = 18 − 21 = −3
Let n^{th} term of the given A.P. be − 81.
$\Rightarrow -81 = a + (n-1)d$
$\Rightarrow -81 = 21 + (n-1) \times (-3)$
$\Rightarrow -102 = (n-1) \times (-3)$
$\Rightarrow n - 1 = 34$
$\Rightarrow n = 35$
Thus, the 35^{th} term of the given A.P. is − 81.
Let p^{th} term of this A.P. be 0.
$\Rightarrow 21 + (p-1) \times (-3) = 0$

$\Rightarrow 21 - 3p + 3 = 0$
$\Rightarrow 3p = 24$
$\Rightarrow p = 8$
Thus, 8^{th} term of this A.P. is 0.

Question 18.
An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find the 31^{st} term.

Solution:
For a given A.P.,
Number of terms, n = 60

First term, a = 7
Last term, l = 125
$\Rightarrow t_{60} = 125$
$\Rightarrow a + 59d = 125$
$\Rightarrow 7 + 59d = 125$
$\Rightarrow 59d = 118$
$\Rightarrow d = 2$
Hence, $t_{31} = a + 30d = 7 + 30(2) = 7 + 60 = 67$

Question 19.
The sum of the 4^{th} and the 8^{th} terms of an A.P. is 24 and the sum of the sixth term and the tenth term is 34. Find the first three terms of the A.P.

Solution:
Let 'a' be the first term and 'd' be the common difference of the given A.P.
$t_4 + t_8 = 24$ (given)
$\Rightarrow (a + 3d) + (a + 7d) = 24$
$\Rightarrow 2a + 10d = 24$
$\Rightarrow a + 5d = 12$ ….(i)

And,
$t_6 + t_{10} = 34$ (given)
$\Rightarrow (a + 5d) + (a + 9d) = 34$
$\Rightarrow 2a + 14d = 34$
$\Rightarrow a + 7d = 17$ ….(ii)

Subtracting (i) from (ii), we get
$2d = 5$

Question 20.
If the third term of an A.P. is 5 and the seventh terms is 9, find the 17^{th} term.

Solution:
Let 'a' be the first term and 'd' be the common difference of the given A.P.
Now, $t_3 = 5$ (given)
$\Rightarrow a + 2d = 5$ ....(i)
And,
$t_7 = 9$ (given)
$\Rightarrow a + 6d = 9$ ....(ii)

Subtracting (i) from (ii), we get
$4d = 4$
$\Rightarrow d = 1$
$\Rightarrow a + 2(1) = 5$
$\Rightarrow a = 3$
Hence, 17^{th} term $= t_{17} = a + 16d = 3 + 16(1) = 19$

Exercise 10B

Question 1.
In an A.P., ten times of its tenth term is equal to thirty times of its 30th term. Find its 40th term.

Solution:

The general term of an A.P. is given by
$t_n = a + (n - 1)d$
Given,
$10 \times t_{10} = 30 \times t_{30}$
$\Rightarrow 10 \times (a + 9d) = 30 \times (a + 29d)$
$\Rightarrow a + 9d = 3 \times (a + 29d)$
$\Rightarrow a + 9d = 3a + 87d$
$\Rightarrow 2a + 78d = 0$
$\Rightarrow a + 39d = 0$
$\Rightarrow a = -39d$
Now, $t_{40} = a + 39d = -39d + 39d = 0$

Question 2.
How many two-digit numbers are divisible by 3?

Solution:

The two-digit numbers divisible by 3 are as follows:
12, 15, 18, 21, ……., 99
Clearly, this forms an A.P. with first term, $a = 12$
and common difference, $d = 3$
Last term = n^{th} term = 99
The general term of an A.P. is given by
$t_n = a + (n - 1)d$
$\Rightarrow 99 = 12 + (n - 1)(3)$
$\Rightarrow 99 = 12 + 3n - 3$
$\Rightarrow 90 = 3n$
$\Rightarrow n = 30$
Thus, 30 two-digit numbers are divisible by 3.

Question 3.
Which term of A.P. 5, 15, 25 ………… will be 130 more than its 31st term?

Solution:

The given A.P. is 5, 15, 25, …..
Here, $a = 5$ and $d = 15 - 5 = 10$
Now, $t_{31} = a + 30d = 5 + 30 \times 10 = 5 + 300 = 305$
Let the required term be n^{th} term.
$\therefore\ t_n - t_{31} = 130$
$\Rightarrow [a + (n - 1)d] - 305 = 130$
$\Rightarrow 5 + (n - 1)(10) = 435$
$\Rightarrow (n - 1)(10) = 430$
$\Rightarrow n - 1 = 43$
$\Rightarrow n = 44$
Thus, required term = 44^{th} term

Question 4.
Find the value of p, if x, 2x + p and 3x + 6 are in A.P

Solution:

Since x, 2x + p and 3x + 6 are in A.P., we have

$(2x + p) - x = (3x + 6) - (2x + p)$

$\Rightarrow 2x + p - x = 3x + 6 - 2x - p$

$\Rightarrow x + p = x + 6 - p$

$\Rightarrow p + p = x - x + 6$

$\Rightarrow 2p = 6$

$\Rightarrow p = 3$

Question 5.
If the 3rd and the 9th terms of an arithmetic progression are 4 and -8 respectively, Which term of it is zero?

Solution:

For an A.P.,

$t_3 = 4$

$\Rightarrow a + 2d = 4 \quad ....(i)$

$t_9 = -8$

$\Rightarrow a + 8d = -8 \quad ....(ii)$

Subtracting (i) from (ii), we get

$6d = -12$

$\Rightarrow d = -2$

Substituting $d = -2$ in (i), we get

$a + 2(-2) = 4$

$\Rightarrow a - 4 = 4$

$\Rightarrow a = 8$

$\Rightarrow$ General term $= t_n = 8 + (n - 1)(-2)$

Let p^{th} term of this A.P. be 0.

$\Rightarrow 8 + (p - 1) \times (-2) = 0$

$\Rightarrow 8 - 2p + 2 = 0$

$\Rightarrow 10 - 2p = 0$

$\Rightarrow 2p = 10$

$\Rightarrow p = 5$

Thus, 5^{th} term of this A.P. is 0.

Question 6.
How many three-digit numbers are divisible by 87?

Solution:

The three-digit numbers divisible by 87 are as follows:
174, 261,......, 957
Clearly, this forms an A.P. with first term, a = 174
and common difference, d = 87
Last term = n^{th} term = 957
The general term of an A.P. is given by
$t_n = a + (n-1)d$
$\Rightarrow 957 = 174 + (n-1)(87)$
$\Rightarrow 783 = (n-1) \times 87$
$\Rightarrow 9 = n - 1$
$\Rightarrow n = 10$
Thus, 10 three-digit numbers are divisible by 87.

Question 7.
For what value of n, the nth term of A.P 63, 65, 67, …….. and nth term of A.P. 3, 10, 17,…….. are equal to each other?

Solution:

For an A.P. 63, 65, 67, ..., we have a = 63 and d = 65 − 63 = 2
n^{th} term = $t_n = 63 + (n-1) \times 2$

For an A.P. 3, 10, 17, ..., we have a' = 3 and d' = 10 − 3 = 7
n^{th} term = $t'_n = 3 + (n-1) \times 7$

The two A.P.s will have equal n^{th} terms is
$t_n = t'_n$
$\Rightarrow 63 + (n-1) \times 2 = 3 + (n-1) \times 7$
$\Rightarrow 63 + 2n - 2 = 3 + 7n - 7$
$\Rightarrow 61 + 2n = 7n - 4$
$\Rightarrow 5n = 65$
$\Rightarrow n = 13$

Question 8.
Determine the A.P. Whose 3rd term is 16 and the 7th term exceeds the 5th term by 12.

Solution:

For given A.P.,
$t_3 = a + 2d = 16 \quad \ldots.(i)$
Now,
$t_7 - t_5 = 12$
$\Rightarrow (a + 6d) - (a + 4d) = 12$
$\Rightarrow 2d = 12$
$\Rightarrow d = 6$
Substituting the value of d in (i), we get
$a + 2 \times 6 = 16$
$\Rightarrow a + 12 = 16$
$\Rightarrow a = 4$
Thus, the required A.P. $= a,\ a + d,\ a + 2d,\ a + 3d, \ldots..$
$= 4, 10, 16, 22, \ldots..$

Question 9.
If numbers n – 2, 4n – 1 and 5n + 2 are in A.P. find the value of n and its next two terms.

Solution:

Since $(n - 2)$, $(4n - 1)$ and $(5n + 2)$ are in A.P., we have
$(4n - 1) - (n - 2) = (5n + 2) - (4n - 1)$
$\Rightarrow 4n - 1 - n + 2 = 5n + 2 - 4n + 1$
$\Rightarrow 3n + 1 = n + 3$
$\Rightarrow 2n = 2$
$\Rightarrow n = 1$
So, the given numbers are $-1, 3, 7$
$\Rightarrow a = -1$ and $d = 3 - (-1) = 4$
Hence, the next two terms are $(7 + 4)$ and $(7 + 2 \times 4)$
i.e. 11 and 15.

Question 10.

Determine the value of k for which $k^2 + 4k + 8$, $2k^2 + 3k + 6$ and $3k^2 + 4k + 4$ are in A.P

Solution:

Since $(k^2 + 4k + 8)$, $(2k^2 + 3k + 6)$ and $(3k^2 + 4k + 4)$ are in A.P., we have

$(2k^2 + 3k + 6) - (k^2 + 4k + 8) = (3k^2 + 4k + 4) - (2k^2 + 3k + 6)$

$\Rightarrow 2k^2 + 3k + 6 - k^2 - 4k - 8 = 3k^2 + 4k + 4 - 2k^2 - 3k - 6$

$\Rightarrow k^2 - k - 2 = k^2 + k - 2$

$\Rightarrow 2k = 0$

$\Rightarrow k = 0$

Question 11.

If a, b and c are in A.P show that:

(i) 4a, 4b and 4c are in A.P

(ii) a + 4, b + 4 and c + 4 are in A.P.

Solution:

a, b and c are in A.P.

$\Rightarrow b - a = c - b$

$\Rightarrow 2b = a + c$

(i) Given terms are 4a, 4b and 4c

Now, $4b - 4a = 2(2b - 2a)$

$= 2(a + c - 2a)$

$= 2(c - a)$

And, $4c - 4b = 2(2c - 2b)$

$= 2(2c - a - c)$

$= 2(c - a)$

Since $4b - 4a = 4c - 4b$, the given terms are in A.P.

(ii) Given terms are $(a + 4)$, $(b + 4)$ and $(c + 4)$

Now, $(b + 4) - (a + 4) = b - a$

$= \frac{a + c}{2} - a$

$= \frac{a + c - 2a}{2}$

$= \frac{c - a}{2}$

a, b and c are in A.P.

$\Rightarrow b - a = c - b$

$\Rightarrow 2b = a + c$

(i) Given terms are 4a, 4b and 4c

$$\begin{aligned} \text{Now, } 4b - 4a &= 2(2b - 2a) \\ &= 2(a + c - 2a) \\ &= 2(c - a) \end{aligned}$$

$$\begin{aligned} \text{And, } 4c - 4b &= 2(2c - 2b) \\ &= 2(2c - a - c) \\ &= 2(c - a) \end{aligned}$$

Since $4b - 4a = 4c - 4b$, the given terms are in A.P.

(ii) Given terms are $(a + 4)$, $(b + 4)$ and $(c + 4)$

$$\begin{aligned} \text{Now, } (b + 4) - (a + 4) &= b - a \\ &= \frac{a + c}{2} - a \\ &= \frac{a + c - 2a}{2} \\ &= \frac{c - a}{2} \end{aligned}$$

$$\begin{aligned} \text{And, } (c + 4) - (b + 4) &= c - b \\ &= c - \frac{a + c}{2} \\ &= \frac{2c - a - c}{2} \\ &= \frac{c - a}{2} \end{aligned}$$

Since $(b + 4) - (a + 4) = (c + 4) - (b + 4)$, the given terms are in A.P.

Question 12.

An A.P consists of 57 terms of which 7th term is 13 and the last term is 108. Find the 45th term of this A.P.

Solution:

Number of terms = n = 57

$t_7 = 13$

$\Rightarrow a + 6d = 13 \quad ....(i)$

Last term = $t_{57} = 108$

$\Rightarrow a + 56d = 108 \quad ....(ii)$

Subtracting (i) from (ii), we get

$50d = 95$

$\Rightarrow d = \frac{95}{50}$

$\Rightarrow d = \frac{19}{10}$

Substituting value of d in (i), we get

$a + 6 \times \frac{19}{10} = 13$

$\Rightarrow a + \frac{57}{5} = 13$

$\Rightarrow a = 13 - \frac{57}{5} = \frac{65 - 57}{5} = \frac{8}{5}$

$\Rightarrow$ General term = $t_n = \frac{8}{5} + (n - 1) \times \frac{19}{10}$

$\Rightarrow t_{45} = \frac{8}{5} + 44 \times \frac{19}{10} = \frac{8}{5} + \frac{418}{5} = \frac{426}{5} = 85.2$

Question 13.

4th term of an A.P is equal to 3 times its first term and 7th term exceeds twice the 3rd time by I. Find the first term and the common difference.

Solution:

The general term of an AP is given by $t_n = a + (n - 1)d$

Now, $t_4 = 3 \times a$

$\Rightarrow a + 3d = 3a$

$\Rightarrow 2a - 3d = 0 \; ....(i)$

Next, $t_7 - 2 \times t_3 = 1$

$\Rightarrow a + 6d - 2(a + 2d) = 1$

$\Rightarrow a + 6d - 2a - 4d = 1$

$\Rightarrow -a + 2d = 1 \quad ....(ii)$

Multiplying (ii) by 2, we get

$-2a + 4d = 2 \quad ....(iii)$

Adding equations (i) and (iii), we get

$d = 2$

Substituting the value of d in (ii), we get

$-a + 2 \times 2 = 1$

$\Rightarrow -a + 4 = 1$

$\Rightarrow a = 3$

Hence, $a = 3$ and $d = 2$

Question 14.

The sum of the 2nd term and the 7th term of an A.P is 30. If its 15th term is 1 less than twice of its 8th term, find the A.P

Solution:

The general term of an AP is given by $t_n = a + (n - 1)d$

Now, $t_2 + t_7 = 30$

$\Rightarrow (a + d) + (a + 6d) = 30$

$\Rightarrow 2a + 7d = 30 \quad ....(i)$

Next, $2 \times t_8 - t_{15} = 1$

$\Rightarrow 2 \times (a + 7d) - (a + 14d) = 1$

$\Rightarrow 2a + 14d - a - 14d = 1$

$\Rightarrow a = 1$

Substituting the value of a in (i), we get

$2 \times 1 + 7d = 30$

$\Rightarrow 7d = 28$

$\Rightarrow d = 4$

Thus, required A.P. $= a, a + d, a + 2d, a + 3d, ...$

$= 1, 5, 9, 13, ...$

Question 15.
In an A.P, if mth term is n and nth term is m, show that its rth term is (m + n − r)

Solution:

For an A.P.,

$t_m = n$

$\Rightarrow a + (m-1)d = n \quad ....(i)$

And, $t_n = m$

$\Rightarrow a + (n-1)d = m \quad ....(ii)$

Subtracting (i) from (ii), we get

$(n-1)d - (m-1)d = m - n$

$\Rightarrow nd - d - md + d = m - n$

$\Rightarrow d(n-m) = m - n$

$\Rightarrow -d(m-n) = m - n$

$\Rightarrow d = -1$

Substituting $d = -1$ in equation (i), we get

$a + (m-1)(-1) = n$

$\Rightarrow a - m + 1 = n$

$\Rightarrow a = m + n - 1$

Now, $t_r = a + (r-1)d$

$= (m + n - 1) + (r-1)(-1)$

$= m + n - 1 - r + 1$

$= m + n - r$

Question 16.
Which term of the A.P 3, 10, 17, ………. Will be 84 more than its 13th term?

Solution:

The given A.P. is 3, 10, 17, ...

Here, $a = 3$ and $d = 10 - 3 = 7$

Now,

$t_{13} = a + 12d = 3 + 12 \times 7 = 3 + 84 = 87$

Let the required term be n^{th} term.

$\therefore t_n - t_{13} = 84$

$\Rightarrow [a + (n-1)d] - 87 = 84$

$\Rightarrow 3 + (n-1) \times 7 = 171$

$\Rightarrow (n-1)\times 7 = 168$
$\Rightarrow n-1 = 24$
$\Rightarrow n = 25$
Thus, required term = 25^{th} term

Exercise 10 C

Question 1.
Find the sum of the first 22 terms of the A.P.: 8, 3, -2, ………..

Solution:

The given A.P. is 8, 3, − 2, ……
Here, $a = 8$, $d = 3 - 8 = -5$ and $n = 22$

$$\therefore\ S = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{22}{2}[2\times 8 + (22-1)\times(-5)]$$
$$= 11[16 + 21\times(-5)]$$
$$= 11[16 - 105]$$
$$= 11\times(-89)$$
$$= -979$$

Question 2.
How many terms of the A.P. :
24, 21, 18, ……… must be taken so that their sum is 78?

Solution:

Let the number of terms taken be n.
The given A.P. is 24, 21, 18, ……
Here, $a = 24$ and $d = 21 - 24 = -3$

$$S = \frac{n}{2}[2a + (n-1)d]$$
$$\Rightarrow 78 = \frac{n}{2}[2\times 24 + (n-1)\times(-3)]$$
$$\Rightarrow 78 = \frac{n}{2}[48 - 3n + 3]$$
$$\Rightarrow 156 = n[51 - 3n]$$
$$\Rightarrow 156 = 51n - 3n^2$$

$\Rightarrow 3n^2 - 51n + 156 = 0$
$\Rightarrow n^2 - 17n + 52 = 0$
$\Rightarrow n^2 - 13n - 4n + 52 = 0$
$\Rightarrow n(n-13) - 4(n-13) = 0$
$\Rightarrow (n-13)(n-4) = 0$
$\Rightarrow n = 13$ or $n = 4$
$\therefore$ Required number of terms = 4 or 13

Question 3.
Find the sum of 28 terms of an A.P. whose nth term is 8n – 5.

Solution:

n^{th} term of an A.P. = $t_n = 8n - 5$
Let a be the first term and d be the common difference of this A.P.
Then,
$a = t_1 = 8 \times 1 - 5 = 8 - 5 = 3$
$t_2 = 8 \times 2 - 5 = 16 - 5 = 11$
$\therefore d = t_2 - t_1 = 11 - 3 = 8$
The sum of n terms of an A.P. = $S = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow$ Sum of 28 terms of an A.P. = $\frac{28}{2}[2 \times 3 + 27 \times 8]$
$= 14[6 + 216]$
$= 14 \times 222$
$= 3108$

Question 4(i).
Find the sum of all odd natural numbers less than 50

Solution:

Odd natural numbers less than 50 are as follows:
1, 3, 5, 7, 9, ..., 49
Now, $3 - 1 = 2$, $5 - 3 = 2$ and so on.
Thus, this forms an A.P. with first term $a = 1$,
common difference $d = 2$ and last term $l = 49$
Now, $l = a + (n-1)d$

$\Rightarrow 49 = 1 + (n - 1) \times 2$

$\Rightarrow 48 = (n - 1) \times 2$

$\Rightarrow 24 = n - 1$

$\Rightarrow n = 25$

Sum of first n terms = $S = \frac{n}{2}[a + l]$

$\Rightarrow$ Sum of odd natural numbers less than 50 $= \frac{25}{2}[1 + 49]$

$= \frac{25}{2} \times 50$

$= 25 \times 25$

$= 625$

Question 4(ii).

Find the sum of first 12 natural numbers each of which is a multiple of 7.

Solution:

First 12 natural numbers which are multiple of 7 are as follows:

7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84

Clearly, this forms an A.P. with first term $a = 7$,
common difference $d = 7$ and last term $l = 84$

Sum of first n terms = $S = \frac{n}{2}[a + l]$

$\Rightarrow$ Sum of first 12 natural numbers which are multiple of 7 $= \frac{12}{2}[7 + 84]$

$= 6 \times 91$

$= 546$

Question 5.

Find the sum of first 51 terms of an A.P. whose 2nd and 3rd terms are 14 and 18 respectively.

Solution:

Given, $t_2 = 14$ and $t_3 = 18$

$\Rightarrow d = t_3 - t_2 = 18 - 14 = 4$

Now, $t_2 = 14$

$\Rightarrow a + d = 14$

$\Rightarrow a + 4 = 14$

$\Rightarrow a = 10$

Sum of n terms of an A.P. $= \frac{n}{2}\left[2a + (n-1)d\right]$

$\therefore$ Sum of first 51 terms of an A.P. $= \frac{51}{2}[2 \times 10 + 50 \times 4]$

$= \frac{51}{2}[20 + 200]$

$= \frac{51}{2} \times 220$

$= 51 \times 110$

$= 5610$

Question 6.

The sum of first 7 terms of an A.P is 49 and that of first 17 terms of it is 289. Find the sum of first n terms

Solution:

Sum of first 7 terms of an A.P = 49

$\Rightarrow \frac{7}{2}[2a + 6d] = 49$

$\Rightarrow \frac{7}{2} \times 2[a + 3d] = 49$

$\Rightarrow 7[a + 3d] = 49$

$\Rightarrow a + 3d = 7 \quad \ldots.(i)$

Sum of first 17 terms of A.P. = 289

$\Rightarrow \frac{17}{2}[2a + 16d] = 289$

$\Rightarrow \frac{17}{2} \times 2[a + 8d] = 289$

$\Rightarrow 17[a + 8d] = 289$

$\Rightarrow a + 8d = 17 \quad \ldots.(ii)$

Subtracting (i) from (ii), we get

$5d = 10 \Rightarrow d = 2$

Substituting d = 2 in (i), we get

$a + 3 \times 2 = 7$

$\Rightarrow a + 6 = 7 \Rightarrow a = 1$

$$\therefore \text{Sum of first n terms} = \frac{n}{2}[2 \times 1 + (n-1)2]$$
$$= \frac{n}{2}[2 + 2n - 2]$$
$$= \frac{n}{2} \times 2n$$
$$= n^2$$

Question 7.

The first term of an A.P is 5, the last term is 45 and the sum of its terms is 1000. Find the number of terms and the common difference of the A.P.

Solution:

First term a = 5

Last term l = 45

Sum of terms = 1000

Let there be n terms in this A.P.

Now, sum of first n terms $= \frac{n}{2}[a + l]$

$\Rightarrow 1000 = \frac{n}{2}[5 + 45]$

$\Rightarrow 2000 = n \times 50$

$\Rightarrow n = 40$

$l = a + (n-1)d$

$\Rightarrow 45 = 5 + (40 - 1)d$

$\Rightarrow 40 = 39d$

$\Rightarrow d = \frac{40}{39}$

Hence, numbers of terms are 40 and common difference is $\frac{40}{39}$.

Question 8.
Find the sum of all natural numbers between 250 and 1000 which are divisible by 9.

Solution:

Natural numbers between 250 and 1000 which are divisible by 9 are as follows:
252, 261, 270, 279, ...,999

Clearly, this forms an A.P. with first term a = 252,
common difference d = 9 and last term l = 999

$l = a + (n - 1)d$
$\Rightarrow 999 = 252 + (n - 1) \times 9$
$\Rightarrow 747 = (n - 1) \times 9$
$\Rightarrow n - 1 = 83$
$\Rightarrow n = 84$

Sum of first n terms = $S = \frac{n}{2}[a + l]$
$\Rightarrow$ Sum of natural numbers between 250 and 1000 which are divisible by 9
$= \frac{84}{2}[252 + 999]$
$= 42 \times 1251$
$= 52542$

Question 9.
The first and the last terms of an A.P. are 34 and 700 respectively. If the common difference is 18, how many terms are there and what is their sum?

Solution:

Let there be n terms in this A.P.
First term a = 34
Common difference d = 18
Last term l = 700
$\Rightarrow a + (n - 1)d = 700$
$\Rightarrow 34 + (n - 1) \times 18 = 700$
$\Rightarrow (n - 1) \times 18 = 666$
$\Rightarrow n - 1 = 37$
$\Rightarrow n = 38$

$$\text{Sum of first n terms} = \frac{n}{2}[a + l] = \frac{38}{2}[34 + 700] = 19 \times 734 = 13946$$

Question 10.
In an A.P, the first term is 25, nth term is -17 and the sum of n terms is 132. Find n and the common difference.

Solution:

First term $a = 25$

n^{th} term $= -17 \Rightarrow$ Last term $l = -17$

Sum of n terms $= 132$

$\Rightarrow \frac{n}{2}[a + l] = 132$

$\Rightarrow n(25 - 17) = 264$

$\Rightarrow n \times 8 = 264$

$\Rightarrow n = 33$

Now, $l = -17$

$\Rightarrow a + (n - 1)d = -17$

$\Rightarrow 25 + 32d = -17$

$\Rightarrow 32d = -42$

$\Rightarrow d = -\frac{42}{32}$

$\Rightarrow d = -\frac{21}{16}$

Question 11.
If the 8th term of an A.P is 37 and the 15th term is 15 more than the 12th term, find the A.P. Also, find the sum of first 20 terms of A.P.

Solution:

For an A.P.

$t_8 = 37$

$\Rightarrow a + 7d = 37 \quad \ldots.(i)$

Also, $t_{15} - t_{12} = 15$

$\Rightarrow (a + 14d) - (a + 11d) = 15$

$\Rightarrow a + 14d - a - 11d = 15$

$\Rightarrow 3d = 15$

$\Rightarrow d = 5$
Substituting $d = 5$ in (i), we get
$a + 7 \times 5 = 37$
$\Rightarrow a + 35 = 37$
$\Rightarrow a = 2$

$\therefore$ Required A.P. $= a,\ a + d,\ a + 2d,\ a + 3d, \ldots..$
$= 2, 7, 12, 17, \ldots\ldots..$

Sum of first 20 terms of this A.P. $= \frac{20}{2}[2 \times 2 + 19 \times 5]$
$= 10[4 + 95]$
$= 10 \times 99$
$= 990$

Question 12.
Find the sum of all multiples of 7 between 300 and 700.

Solution:

Numbers between 300 and 700 which are multiple of 7 are as follows:
301, 308, 315, 322,……, 693

Clearly, this forms an A.P. with first term $a = 301$,
common difference $d = 7$ and last term $l = 693$
$l = a + (n - 1)d$
$\Rightarrow 693 = 301 + (n - 1) \times 7$
$\Rightarrow 392 = (n - 1) \times 7$
$\Rightarrow n - 1 = 56$
$\Rightarrow n = 57$

Sum of first n terms $= S = \frac{n}{2}[a + l]$
$\Rightarrow$ Required sum $= \frac{57}{2}[301 + 693]$
$= \frac{57}{2} \times 994$
$= 57 \times 497$
$= 28329$

Question 13.

The sum of n natural numbers is $5n^2 + 4n$. Find its 8^{th} term

Solution:

Sum of n natural numbers = $S_n = 5n^2 + 4n$

$\Rightarrow$ Sum of $(n-1)$ natural numbers = $S_{n-1} = 5(n-1)^2 + 4(n-1)$

$= 5(n^2 + 1 - 2n) + 4n - 4$

$= 5n^2 + 5 - 10n + 4n - 4$

$= 5n^2 - 6n + 1$

n^{th} term $= S_n - S_{n-1} = 5n^2 + 4n - 5n^2 + 6n - 1 = 10n - 1$

$\Rightarrow 8^{th}$ term $= t_8 = 10 \times 8 - 1 = 80 - 1 = 79$

Question 14.

The fourth term of an A.P. is 11 and the term exceeds twice the fourth term by 5 the A.P and the sum of first 50 terms

Solution:

For an A.P.

$t_4 = 11$

$\Rightarrow a + 3d = 11 \quad ....(i)$

Also, $t_8 - 2t_4 = 5$

$\Rightarrow (a + 7d) - 2 \times 11 = 5$

$\Rightarrow a + 7d - 22 = 5$

$\Rightarrow a + 7d = 27 \quad ....(ii)$

Subtracting (i) from (ii), we get

$4d = 16$

$\Rightarrow d = 4$

Substituting $d = 4$ in (i), we get

$a + 3 \times 4 = 11$

$\Rightarrow a + 12 = 11$

$\Rightarrow a = -1$

$\therefore$ Required A.P. = $a, a+d, a+2d, a+3d, \ldots$

$= -1, 3, 7, 11, \ldots$

Sum of first 50 terms of this A.P. $= \frac{50}{2}[2\times(-1)+49\times 4]$

$= 25[-2+196]$

$= 25\times 194$

$= 4850$

Exercise 10 D

Question 1.
Find three numbers in A.P. whose sum is 24 and whose product is 440.

Solution:

Let the three numbers in A.P. be $a-d$, a and $a+d$.

$\therefore (a-d)+a+(a+d) = 24$

$\Rightarrow 3a = 24$

$\Rightarrow a = 8 \quad \ldots(i)$

Also, $(a-d)\times a\times(a+d) = 440$

$\Rightarrow (a^2-d^2)\times a = 440$

$\Rightarrow (8^2-d^2)\times 8 = 440 \quad \ldots[\text{From (i)}]$

$\Rightarrow 64-d^2 = 55$

$\Rightarrow d^2 = 9$

$\Rightarrow d = \pm 3$

When $a = 8$ and $d = 3$

Required terms = $a-d$, a and $a+d$

$= 8-3, 8, 8+3$

$= 5, 8, 11$

When $a = 8$ and $d = -3$

Required terms = $a-d$, a and $a+d$

$= 8-(-3), 8, 8+(-3)$

$= 11, 8, 5$

Question 2.

The sum of three consecutive terms of an A.P. is 21 and the slim of their squares is 165. Find these terms.

Solution:

Let the three consecutive terms in A.P. be a – d, a and a + d.

$\therefore (a - d) + a + (a + d) = 21$

$\Rightarrow 3a = 21$

$\Rightarrow a = 7 \quad \ldots.(i)$

Also, $(a - d)^2 + a^2 + (a + d)^2 = 165$

$\Rightarrow a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 165$

$\Rightarrow 3a^2 + 2d^2 = 165$

$\Rightarrow 3 \times (7)^2 + 2d^2 = 165 \quad \ldots.[\text{From (i)}]$

$\Rightarrow 3 \times 49 + 2d^2 = 165$

$\Rightarrow 147 + 2d^2 = 165$

$\Rightarrow 2d^2 = 18$

$\Rightarrow d^2 = 9$

$\Rightarrow d = \pm 3$

When a = 7 and d = 3

Required terms = a – d, a and a + d

= 7 – 3, 7, 7 + 3

= 4, 7, 10

When a = 7 and d = –3

Required terms = a – d, a and a + d

= 7 – (–3), 7, 7 + (–3)

= 10, 7, 4

Question 3.
The angles of a quadrilateral are in A.P. with common difference 20°. Find its angles.

Solution:

Let the four angles of a quadrilateral in A.P. be a, $a+20^\circ, a+40^\circ$ and $a+60^\circ$

$\therefore\ a+(a+20^\circ)+(a+40^\circ)+(a+60^\circ)=360^\circ$ ....[Angle sum property]

$\Rightarrow 4a+120^\circ=360^\circ$

$\Rightarrow 4a=240^\circ$

$\Rightarrow a=60^\circ$ ....(i)

Thus, angles of a quadrilateral are $= a,\ a+20^\circ, a+40^\circ$ and $a+60^\circ$

$= 60^\circ, 80^\circ, 100^\circ$ and 120°

Question 4.
Divide 96 into four parts which are in A.P. and the ratio between product of their means to product of their extremes is 15 : 7.

Solution:

Let the four parts be $(a-3d),(a-d),(a+d)$ and $(a+3d)$.

Then, $(a-3d)+(a-d)+(a+d)+(a+3d)=96$

$\Rightarrow 4a=96$

$\Rightarrow a=24$

It is given that

$$\frac{(a-d)(a+d)}{(a-3d)(a+3d)}=\frac{15}{7}$$

$$\Rightarrow \frac{a^2-d^2}{a^2-9d^2}=\frac{15}{7}$$

$$\Rightarrow \frac{576-d^2}{576-9d^2}=\frac{15}{7}$$

$\Rightarrow 4032-7d^2=8640-135d^2$

$\Rightarrow 128d^2=4608$

$\Rightarrow d^2=36$

$\Rightarrow d=\pm 6$

When $a = 24, d = 6$
$a - 3d = 24 - 3(6) = 6$
$a - d = 24 - 6 = 18$
$a + d = 24 + 6 = 30$
$a + 3d = 24 + 3(6) = 42$

When $a = 24, d = -6$
$a - 3d = 24 - 3(-6) = 42$
$a - d = 24 - (-6) = 30$
$a + d = 24 + (-6) = 18$
$a + 3d = 24 + 3(-6) = 6$

Thus, the four parts are (6, 18, 30, 42) or (42, 30, 18, 6).

Question 5.
Find five numbers in A.P. whose sum is $12\frac{1}{2}$ and the ratio of the first to the last terms is 2: 3.

Solution:

Let the five numbers in A.P. be $(a-2d), (a-d), a, (a+d)$ and $(a+2d)$.

Then, $(a-2d)+(a-d)+a+(a+d)+(a+2d) = 12\frac{1}{2}$

$\Rightarrow 5a = \frac{25}{2}$

$\Rightarrow a = \frac{5}{2}$

It is given that

$\frac{a-2d}{a+2d} = \frac{2}{3}$

$\Rightarrow 3a - 6d = 2a + 4d$

$\Rightarrow a = 10d$

$\Rightarrow \frac{5}{2} = 10d$

$\Rightarrow d = \frac{1}{4}$

$\Rightarrow a = \frac{5}{2}$ and $d = \frac{1}{4}$

Thus, we have

$a - 2d = \frac{5}{2} - 2 \times \frac{1}{4} = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

$a - d = \frac{5}{2} - \frac{1}{4} = \frac{10 - 1}{4} = \frac{9}{4}$

$a = \frac{5}{2}$

$a + d = \frac{5}{2} + \frac{1}{4} = \frac{10 + 1}{4} = \frac{11}{4}$

$a + 3d = \frac{5}{2} + 2 \times \frac{1}{4} = \frac{5}{2} + \frac{1}{2} = \frac{6}{2} = 3$

Thus, the five numbers in A.P. $= 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}$ and 3

$= 2, 2.25, 2.5, 2.75$ and 3

Question 6.

Split 207 into three parts such that these parts are in A.P. and the product of the two smaller parts is 4623.

Solution:

Let the three parts in A.P. be $(a - d)$, a and $(a + d)$.

Then, $(a - d) + a + (a + d) = 207$

$\Rightarrow 3a = 207$

$\Rightarrow a = 69$

It is given that

$(a - d) \times a = 4623$

$\Rightarrow (69 - d) \times 69 = 4623$

$\Rightarrow 69 - d = 67$

$\Rightarrow d = 2$

$\Rightarrow a = 69$ and $d = 2$

Thus, we have

$a - d = 69 - 2 = 67$

$a = 69$

$a + d = 69 + 2 = 71$

Thus, the three parts in A.P are 67, 69 and 71.

Question 7.
The sum of three numbers in A.P. is 15 the sum of the squares of the extreme is 58. Find the numbers.

Solution:

Let the three numberss in A.P. be $(a-d)$, a and $(a+d)$.

Then, $(a-d)+a+(a+d)=15$

$\Rightarrow 3a=15$

$\Rightarrow a=5$

It is given that

$(a-d)^2+(a+d)^2=58$

$\Rightarrow a^2+d^2-2ad+a^2+d^2+2ad=58$

$\Rightarrow 2a^2+2d^2=58$

$\Rightarrow 2(a^2+d^2)=58$

$\Rightarrow a^2+d^2=29$

$\Rightarrow 5^2+d^2=29$

$\Rightarrow 25+d^2=29$

$\Rightarrow d^2=4$

$\Rightarrow d=\pm 2$

When $a=5$ and $d=2$,

$a-d=5-2=3$

$a=5$

$a+d=5+2=7$

When $a=5$ and $d=-2$,

$a-d=5-(-2)=7$

$a=5$

$a+d=5+(-2)=3$

Thus, the three numbers in A.P are $(3,5,7)$ or $(7,5,3)$.

Question 8.
Find four numbers in A.P. whose sum is 20 and the sum of whose squares is 120.

Solution:

Let the four numbers in A.P. be $(a-3d), (a-d), (a+d)$ and $(a+3d)$.
Then, $(a-3d)+(a-d)+(a+d)+(a+3d)=20$
$\Rightarrow 4a = 20$
$\Rightarrow a = 5$
It is given that
$(a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2 = 120$
$\Rightarrow a^2+9d^2-6ad+a^2+d^2-2ad+a^2+d^2+2ad+a^2+9d^2+6ad = 120$
$\Rightarrow 4a^2+20d^2 = 120$
$\Rightarrow a^2+5d^2 = 30$
$\Rightarrow 5^2+5d^2 = 30$
$\Rightarrow 25+5d^2 = 30$
$\Rightarrow 5d^2 = 5$
$\Rightarrow d^2 = 1$
$\Rightarrow d = \pm 1$

When $a = 5,\ d = 1$
$a-3d = 5-3(1) = 2$
$a-d = 5-1 = 4$
$a+d = 5+1 = 6$
$a+3d = 5+3(1) = 8$

When $a = 5,\ d = -1$
$a-3d = 5-3(-1) = 8$
$a-d = 5-(-1) = 6$
$a+d = 5+(-1) = 4$
$a+3d = 5+3(-1) = 2$

Thus, the four parts are (2, 4, 6, 8) or (8, 6, 4, 2).

Question 9.

Insert one arithmetic mean between 3 and 13.

Solution:

Arithmetic mean between 3 and 13 $= \frac{3+13}{2} = \frac{16}{2} = 8$

Question 10.

The angles of a polygon are in A.P. with common difference 5°. If the smallest angle is 120°, find the number of sides of the polygon.

Solution:

Let the number of sides of a polygon be n.
The smallest angle = 120° = a
Common difference in angles = d = 5°
Now, in a polygon of n sides, the sum of interior angles = (2n − 4) × 90°

$$\Rightarrow \frac{n}{2}[2 \times 120° + (n-1) \times 5°] = (2n-4) \times 90°$$
$$\Rightarrow \frac{n}{2}[240° + 5n - 5°] = 180n - 360°$$
$$\Rightarrow n[235° + 5n] = 360n - 720°$$
$$\Rightarrow 235n + 5n^2 = 360n - 720$$
$$\Rightarrow 5n^2 - 125n + 720 = 0$$
$$\Rightarrow n^2 - 25n + 144 = 0$$
$$\Rightarrow n^2 - 16n - 9n + 144 = 0$$
$$\Rightarrow n(n-16) - 9(n-16) = 0$$
$$\Rightarrow (n-16)(n-9) = 0$$
$$\Rightarrow n = 16 \text{ or } n = 9$$

Question 11.

$\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.P. Show that : be, ca and ab are also in A.P.

Solution:

$\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.P.

$\Rightarrow \frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$\Rightarrow \frac{a-b}{ab} = \frac{b-c}{bc}$

$\Rightarrow \frac{a-b}{a} = \frac{b-c}{c}$

$\Rightarrow ac - bc = ab - ac$

$\Rightarrow ac + ac = ab + bc$

$\Rightarrow 2ac = ab + bc$

$\Rightarrow 2ca = ab + bc$

$\Rightarrow$ bc, ca and ab are also in A.P.

Question 12.
Insert four A.M.s between 14 and -1.

Solution:

Let the required arithmetic means (A.M.s) between 14 and -1 be A_1, A_2, A_3 and A_4.

$\Rightarrow 14, A_1, A_2, A_3, A_4$ and -1 are in A.P.

$\Rightarrow 14 =$ First term

$\Rightarrow -1 = 6^{th}$ term of this A.P.

$\Rightarrow -1 = 14 + 5d$

$\Rightarrow 5d = -15$

$\Rightarrow d = -3$

$\Rightarrow A_1 = 14 + d = 14 + (-3) = 11$

$A_2 = 14 + 2d = 14 + 2(-3) = 8$

$A_3 = 14 + 3d = 14 + 3(-3) = 5$

$A_4 = 14 + 4d = 14 + 4(-3) = 2$

Hence, required A.M.s between 14 and -1 = 11, 8, 5 and 2

Question 13.

Insert five A.M.s between -12 and 8.

Solution:

Let the required arithmetic means (A.M.s) between -12 and 8 be A_1, A_2, A_3, A_4 and A_5.

$\Rightarrow -12, A_1, A_2, A_3, A_4, A_5$ and 8 are in A.P.

$\Rightarrow -12 =$ First term

$\Rightarrow 8 = 7^{th}$ term of this A.P.

$\Rightarrow 8 = -12 + 6d$

$\Rightarrow 6d = 20$

$\Rightarrow d = \frac{10}{3}$

$$\Rightarrow A_1 = -12 + d = -12 + \frac{10}{3} = \frac{-36+10}{3} = -\frac{26}{3}$$

$$A_2 = -12 + 2d = -12 + \frac{20}{3} = \frac{-36+20}{3} = -\frac{16}{3}$$

$$A_3 = -12 + 3d = -12 + \frac{30}{3} = \frac{-36+30}{3} = -\frac{6}{3}$$

$$A_4 = -12 + 4d = -12 + \frac{40}{3} = \frac{-36+40}{3} = -\frac{4}{3}$$

$$A_5 = -12 + 5d = -12 + \frac{50}{3} = \frac{-36+50}{3} = -\frac{14}{3}$$

Hence, required A.M.s between -12 and $8 = \frac{-26}{3}, \frac{-16}{3}, \frac{-6}{3}, \frac{-4}{3}$ and $\frac{-14}{3}$

Question 14.

Insert six A.M.s between 15 and -15.

Solution:

Let the required arithmetic means (A.M.s) between 15 and -15 be A_1, A_2, A_3, A_4, A_5 and A_6.

$\Rightarrow 15, A_1, A_2, A_3, A_4, A_5, A_6$ and -15 are in A.P.

$\Rightarrow 15 =$ First term

$\Rightarrow -15 = 8^{th}$ term of this A.P.

$\Rightarrow -15 = 15 + 7d$

$\Rightarrow 7d = -30$

$\Rightarrow d = -\frac{30}{7}$

$\Rightarrow A_1 = 15 + d = 15 - \frac{30}{7} = \frac{105 - 30}{7} = \frac{75}{7}$

$A_2 = 15 + 2d = 15 - \frac{60}{7} = \frac{105 - 60}{7} = \frac{45}{7}$

$A_3 = 15 + 3d = 15 - \frac{90}{7} = \frac{105 - 90}{7} = \frac{15}{7}$

$A_4 = 15 + 4d = 15 - \frac{120}{7} = \frac{105 - 120}{7} = \frac{-15}{7}$

$A_5 = 15 + 5d = 15 - \frac{150}{7} = \frac{105 - 150}{7} = \frac{-45}{7}$

$A_6 = 15 + 6d = 15 - \frac{180}{7} = \frac{105 - 180}{7} = \frac{-75}{7}$

Hence, required A.M.s between 15 and $-15 = \frac{75}{7}, \frac{45}{7}, \frac{15}{7}, \frac{-15}{7} \frac{-45}{7}$ and $\frac{-75}{7}$

Exercise 10E

Question 1.

Two cars start together in the same direction from the same place. The first car goes at uniform speed of 10 km hr^{-1} The second car goes at a speed of 8 km h^{-1} in the first hour and thereafter increasing the speed by 0.5 km h^{-1} each succeeding hour. After how many hours will the two cars meet?

Solution:

Let the two cars meet after n hours.
That means the two cars travel the same distance in n hours.
Distance travelled by the 1^{st} car in n hours = $10 \times n$ km
Distance travelled by the 2^{nd} car in n hours = $\frac{n}{2}\left[2 \times 8 + (n-1) \times 0.5\right]$ km

$\Rightarrow 10 \times n = \frac{n}{2}\left[2 \times 8 + (n-1) \times 0.5\right]$
$\Rightarrow 20 = 16 + 0.5n - 0.5$
$\Rightarrow 0.5n = 4.5$
$\Rightarrow n = 9$
Thus, the two cars will meet after 9 hours.

Question 2.
A sum of ₹ 700 is to be paid to give seven cash prizes to the students of a school for their overall academic performance. If the cost of each prize is ₹ 20 less than its preceding prize; find the value of each of the prizes.

Solution:

Total amount of prize = S_n = Rs. 700
Let the value of the first prize be Rs. a.
Number of prizes = n = 7
Let the value of first prize be Rs. a.
Depreciation in next prize = –Rs. 20
We have,

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow 700 = \frac{7}{2}\left[2a + 6(-20)\right]$

$\Rightarrow 700 = \frac{7}{2}\left[2a - 120\right]$

$\Rightarrow 1400 = 14a - 840$
$\Rightarrow 14a = 2240$
$\Rightarrow a = 160$
$\Rightarrow$ Value of 1^{st} prize = Rs. 160
Value of 2^{nd} prize = Rs. $(160 - 20)$ = Rs. 140
Value of 3^{rd} prize = Rs. $(140 - 20)$ = Rs. 120

Value of 4^{th} prize = Rs. (120 − 20) = Rs. 100
Value of 5^{th} prize = Rs. (100 − 20) = Rs. 80
Value of 6^{th} prize = Rs. (80 − 20) = Rs. 60
Value of 7^{th} prize = Rs. (60 − 20) = Rs. 40

Question 3.
An article can be bought by paying ₹ 28,000 at once or by making 12 monthly instalments. If the first instalment paid is ₹ 3,000 and every other instalment is ₹ 100 less than the previous one, find :
(i) amount of instalment paid in the 9th month
(ii) total amount paid in the instalment scheme.

Solution:

Number of instalments = n = 12
First instalment = a = Rs. 3000
Depreciation in instalment = d = −100
(i) Amount of installment paid in the 9^{th} month

$= t_9$
$= a + 8d$
$= 3000 + 8 \times (-100)$
$= 3000 - 800$
= Rs. 2200

(ii) Total amount paid in the installment scheme

$= S_{12}$
$= \frac{12}{2}[2 \times 3000 + 11 \times (-100)]$
$= 6[6000 - 1100]$
$= 6 \times 4900$
= Rs. 29, 400

Question 4.
A manufacturer of TV sets produces 600 units in the third year and 700 units in the 7^{th} year.

Assuming that the production increases uniformly by a fixed number every year, find :
(i) the production in the first year.
(ii) the production in the 10^{th} year.

(iii) the total production in 7 years.

Solution:

Since the production increases uniformly by a fixed number every year, he sequence formed by the production in different years is an A.P.
Let the production in the first year = a
Common difference = Number of units by which the production increases every year = d

We have,

$t_3 = 600$

$\Rightarrow a + 2d = 600 \quad ....(i)$

$t_7 = 700$

$\Rightarrow a + 6d = 700 \quad ....(ii)$

Subtracting (i) from (ii), we get

$4d = 100 \Rightarrow d = 25$

$\Rightarrow a + 2 \times 25 = 600$

$\Rightarrow a = 550$

(i) The production in the first year = 550 TV sets

(ii) Production in the 10^{th} year = $t_{10} = 550 + 9 \times 25 = 775$ TV sets

(iii) Production in 7 years = $S_7 = \frac{7}{2}[2 \times 550 + 6 \times 25]$

$= \frac{7}{2}[1100 + 150]$

$= \frac{7}{2} \times 1250$

$= 4375$ TV sets

Question 5.

Mrs. Gupta repays her total loan of ₹ 1.18,000 by paying instalments every month. If the instalment for the first month is ₹ 1,000 and it increases by ₹ 100 every month, what amount will she pay as the 30^{th} instalment of loan? What amount of loan she still has to pay after the 30^{th} instalment?

Solution:

Total amount of loan = Rs. 1, 18, 000

First installment = a = Rs. 1000

Increase in instalment every month = d = Rs. 100

30^{th} installment = t_{30}

$= a + 29d$

$= 1000 + 29 \times 100$

$= 1000 + 2900$

= Rs. 3900

Now, amount paid in 30 installments = S_{30}

$= \frac{30}{2}[2 \times 1000 + 29 \times 100]$

$= 15[2000 + 2900]$

$= 15 \times 4900$

= Rs. 73, 500

$\therefore$ Amount of loan to be paid after the 30^{th} installments

= Rs. (1, 18, 000 − 73, 500)

= Rs. 44, 500

Question 6.

In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be five times of the class to which the respective section belongs. If there are 1 to 10 classes in the school and each class has three sections, find how many trees were planted by the students?

Solution:

Since each section of each dass plants five times the number of trees as the dass number and there are three sections of each class, we have

Total number of trees planted by the students from class 1 to 10

$= 3[1 \times 5 + 2 \times 5 + 3 \times 5 + \ldots\ldots + 10 \times 5]$

$= 3[5 + 10 + 15 + \ldots\ldots + 50]$

$= 3\left[\frac{10}{2}(2 \times 5 + 9 \times 5)\right]$

$= 3[5(10 + 45)]$

$= 3 \times 5 \times 55$

$= 825$

Hence, 825 trees were planted by students.

Exercise 10F

Question 1.
The 6^{th} term of an A.P. is 16 and the 14^{th} term is 32. Determine the 36^{th} term.

Solution:
Let 'a' be the first term and 'd' be the common difference of the given A.P.
Now, $t_6 = 16$ (given)
$\Rightarrow a + 5d = 16$ ….(i)

And,
$t_{14} = 32$ (given)
$\Rightarrow a + 13d = 32$ ….(ii)

Subtracting (i) from (ii), we get
$8d = 16$
$\Rightarrow d = 2$
$\Rightarrow a + 5(2) = 16$
$\Rightarrow a = 6$

Hence, 36^{th} term $= t_{36} = a + 35d = 6 + 35(2) = 76$

Question 2.
If the third and the 9^{th} terms of an A.P. term is 12 and the last term is 106. Find the 29^{th} term of the A.P.

Solution:
For an A.P.,a
$t_3 = 4$
$\Rightarrow a + 2d = 4$ … (i)
$t_9 = -8$
$\Rightarrow a + 8d = -8$ …. (ii)

Subtracting (i) from (ii), we get
$6d = -12$
$\Rightarrow d = -2$

Substituting $d = -2$ in (i), we get
$a = 2(-2) = 4$
$\Rightarrow a - 4 = 4$
$\Rightarrow a = 8$

$\Rightarrow$ General term $= t_n = 8 + (n - 1)(-2)$
Let p^{th} term of this A.P. be 0.

⇒ 8 + (0 – 1) (-2) = 0
⇒ 8 – 2p + 2 = 0
⇒ 10 – 2p = 0
⇒ 2p = 10
⇒ p = 5
Thus, 5th term of this A.P. is 0.

Question 3.
An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term of the A.P.

Solution:
For a given A.P.,
Number of terms, n = 50

3rd term, t_3 = 12
⇒ a + 2d = 12 ….(i)
Last term, l = 106
⇒ t_{50} = 106
⇒ a + 49d = 106 ….(ii)

Subtracting (i) from (ii), we get
47d = 94
⇒ d = 2
⇒ a + 2(2) = 12
⇒ a = 8
Hence, t_{29} = a + 28d = 8 + 28(2) = 8 + 56 = 64

Question 4.
Find the arithmetic mean of :
(i) -5 and 41
(ii) 3x – 2y and 3x + 2y
(iii) $(m + n)^2$ and $(m – n)^2$

Solution:

(i) Arithmetic mean of -5 and $41 = \frac{-5-41}{2} = \frac{36}{2} = 18$

(ii) Arithmetic mean of $(3x-2y)$ and $(3x-2y) = \frac{3x-2y-3x-2y}{2} = \frac{6x}{2} = 3x$

(iii) Arithmetic mean of $(m-n)^2$ and $(m-n)^2 = \frac{(m-n)^2-(m-n)^2}{2}$

$$= \frac{m^2-n^2-2mn-m^2-n^2-2mn}{2}$$

$$= \frac{2(m^2-n^2)}{2}$$

$$= m^2-n^2$$

Question 5.
Find the sum of first 10 terms of the A.P. 4 + 6 + 8 + ...

Solution:
Here,
First term, a = 4

Common difference, d = 6 − 4 = 2
n = 10

$$\therefore S = \frac{n}{2}[2a-(n-1)d]$$

$$= \frac{10}{2}[2(4)-9(2)]$$

$$= 5[8-18]$$

$$= 5 \times 26$$

$$= 130$$

Question 6.
Find the sum of first 20 terms of an A.P. whose first term is 3 and the last term is 60.

Solution:
Here,
First term, a = 3
Last term, l = 57

n = 20

$$\therefore S = \frac{n}{2}(a - l)$$
$$= \frac{20}{2}(3 - 57)$$
$$= 10 \times 60$$
$$= 600$$

Question 7.
How many terms of the series 18 + 15 + 12 + ……. when added together will give 45 ?

Solution:
Here, we find that
15 – 18 = 12 – 15 = -3
Thus, the given series is an A.P. with first term 18 and common difference -3.
Let the number of term to be added be ‘n’.

$$S_n = \frac{n}{2}[2a - (n - 1)d]$$
$$\Rightarrow 45 = \frac{n}{2}[2(18) - (n - 1)(-3)]$$

$\Rightarrow 90 = n[36 - 3n + 3]$
$\Rightarrow 90 = n[39 - 3n]$
$\Rightarrow 90 = 3n[13 - n]$
$\Rightarrow 30 = 13n - n^2$
$\Rightarrow n^2 - 13n + 30 = 0$
$\Rightarrow n^2 - 10n - 3n + 30 = 0$
$\Rightarrow n(n - 10) - 3(n - 10) = 0$
$\Rightarrow (n - 10)(n - 3) = 0$
$\Rightarrow n - 10 = 0$ or $n - 3 = 0$
$\Rightarrow n = 10$ or $n = 3$
Thus, required number of term to be added is 3 or 10.

Question 8.
The n^{th} term of a sequence is 8 – 5n. Show that the sequence is an A.P.

Solution:
$t_n = 8 - 5n$

Replacing n by (n + 1), we get
$t_{n+1} = 8 - 5(n + 1) = 8 - 5n - 5 = 3 - 5n$
Now,
$t_{n+1} - t_n = (3 - 5n) - (8 - 5n) = -5$

Since, $(t_{n+1} - t_2)$ is independent of n and is therefore a constant.
Hence, the given sequence is an A.P.

Question 9.
The the general term (n^{th} term) and 23^{rd} term of the sequence 3, 1, -1, -3, ……
Solution:
The given sequence is 1, -1, -3, …..

Now,
1 – 3 = -1 – 1 = -3 – (-1) = -2

Hence, the given sequence is an A.P. with first term a = 3 and common difference d = -2.
The general term (n^{th} term) of an A.P. is given by
$t_n = a + (n - 1)d$
$= 3 + (n - 1)(-2)$
$= 3 - 2n + 2$
$= 5 - 2n$
Hence, 23^{rd} term = $t_{23} = 5 - 2(23) = 5 - 46 = -41$

Question 10.
Which term of the sequence 3, 8, 13, …….. is 78 ?

Solution:
The given sequence is 3, 8, 13, …..
Now,
8 – 3 = 13 – 8 = 5

Hence, the given sequence is an A.P. with first term a = 3 and common difference d = 5.
Let the n^{th} term of the given A.P. be 78.
$\Rightarrow 78 = 3 + (n - 1)(5)$
$\Rightarrow 75 = 5n - 5$
$\Rightarrow 5n = 80$
$\Rightarrow n = 16$
Thus, the 16th term of the given sequence is 78.

Question 11.
Is -150 a term of 11, 8, 5, 2, ……… ?

Solution:
The given sequence is 11, 8, 5, 2, …..
Now,
8 – 11 = 5 – 8 = 2 – 5 = -3
Hence, the given sequence is an A.P. with first term a = 11 and common difference d = -

3.
The general term of an A.P. is given by
$t_n = a + (n - 1)d$
$\Rightarrow -150 = 11 + (n - 1)(-5)$
$\Rightarrow -161 = -5n + 5$
$\Rightarrow 5n = 166$
$\Rightarrow n = \frac{166}{5}$

The number of terms cannot be a fraction.
So, clearly, -150 is not a term of the given sequence.

Question 12.
How many two digit numbers are divisible by 3 ?

Solution:
The two-digit numbers divisible by 3 are as follows: 12, 15, 18, 21, …….. 99
Clearly, this forms an A.P. with first term, a = 12
and common difference, d = 3

Last term = n^{th} term= 99
The general term of an A.P. is given by
$t_n = a + (n - 1)d$
$\Rightarrow 99 - 12 + (n - 1)(3)$
$\Rightarrow 99 - 12 + 3n-3$
$\Rightarrow 90 - 3n$
$\Rightarrow n = 30$
Thus, 30 two-digit numbers are divisible by 3.

Question 13.
How many multiples of 4 lie between 10 and 250 ?

Solution:
Numbers between 10 and 250 which are multiple of 4 are as follows: 12, 16, 20, 24,……, 248

Clearly, this forms an A.P. with first term a = 12,
common difference d= 4 and last term l = 248
$l - a + (n - 1)d$
$\Rightarrow 248 - 12 + (n - 1) \times 4$
$\Rightarrow 236 - (n - 1) \times 4$
$\Rightarrow n - 1 = 59$
$\Rightarrow n = 60$
Thus, 60 multiples of 4 lie between 10 and 250.

Question 14.
The sum of the 4^{th} term and the 8^{th} term of an A.P. is 24 and the sum of 6^{th} term and the 10^{th} term is 44. Find the first three terms of the A.P.

Solution:
Given, $t_4 + t_8 = 24$
(a + 3d) + (a + 7d) = 24
= 2a+ 10d = 24
> a + 5d = 12 ….(i)
And,
$t_{62} + t_{10} = 44$
= (a + 5d) + (a + 9d) = 44
= 2a+ 14d = 44
= a + 7 = 22 …(ii)

Subtracting (i) from (ii), we get
2d = 10
= d = 5

Substituting value of din (i), we get
a + 5 × 5 = 12
= a + 25 = 12
= a = -13 = 1^{st} term
a + d = -13 + 5 = -8 = 2^{nd} term
a + 2d = -13 + 2 × 5 = -13 + 10= -3 = 3^{rd} term

Hence, the first three terms of an A.P. are – 13,- 8 and -5.

Question 15.
The sum of first 14 terms of an A.P. is 1050 and its 14^{th} term is 140. Find the 20^{th} term.

Solution:
Let ‘a’ be the first term and ‘d’ be the common difference of the given A.P.
Given,
S_{14}= 1050

$\frac{14}{2}[2a + (14 - 1)d] = 1050$
⇒ 7[2a + 13d] = 1050
⇒ 2a + 13d = 150
⇒ a + 6.5d = 75 ….(i)
And, $t_{14} = 140$

⇒ a + 13d = 140 ….(ii)
Subtracting (i) from (ii), we get

$6.5d = 65$
$\Rightarrow d = 10$
$\Rightarrow a + 13(10) = 140$
$\Rightarrow a = 10$
Thus, 20^{th} term = $t_{20} = 10 + 19d = 10 + 19(10) = 200$

Question 16.
The 25^{th} term of an A.P. exceeds its 9^{th} term by 16. Find its common difference.

Solution:
n^{th} term of an A.P. is given by $t_n = a + (n - 1)d$.
$\Rightarrow t_{25} = a + (25 - 1)d = a + 24d$ and
$t_9 = a + (9 - 1)d = a + 8d$

According to the condition in the question, we get
$t_{25} = t_9 + 16$
$\Rightarrow a + 24d = a + 8d + 16$
$\Rightarrow 16d = 16$
$\Rightarrow d = 1$

Question 17.
For an A.P., show that:
$(m + n)^{th}$ term + $(m - n)^{th}$ term = $2 \times m^{th}$ term

Solution:
Let a and d be the first term and common difference respectively.
$\Rightarrow (m + n)^{th}$ term = $a + (m + n - 1)d$ …. (i) and
$(m - n)^{th}$ term = $a + (m - n - 1)d$ …. (ii)

From (i) + (ii), we get
$(m + n)^{th}$ term + $(m - n)^{th}$ term
$= a + (m + n - 1)d + a + (m - n - 1)d$
$= a + md + nd - d + a + md - nd - d$
$= 2a + 2md - 2d$
$= 2a + (m - 1)2d$
$= 2[a + (m - 1)d]$
$= 2 \times m^{th}$ term
Hence proved.

Question 18.
If the n^{th} term of the A.P. 58, 60, 62,…. is equal to the n^{th} term of the A.P. -2, 5, 12, …., find the value of n.

Solution:
In the first A.P. 58, 60, 62,….
a = 58 and d = 2
$t_n = a + (n - 1)d$
$\Rightarrow t_n = 58 + (n - 1)2$ …. (i)

In the first A.P. -2, 5, 12, ….
a = -2 and d = 7
$t_n = a + (n - 1)d$
$\Rightarrow t_n = -2 + (n - 1)7$ …. (ii)

Given that the n^{th} term of first A.P is equal to the n^{th} term of the second A.P.
$\Rightarrow 58 + (n - 1)2 = -2 + (n - 1)7$ … from (i) and (ii)
$\Rightarrow 58 + 2n - 2 = -2 + 7n - 7$
$\Rightarrow 65 = 5n$
$\Rightarrow n = 15$

Question 19.
Which term of the A.P. 105, 101, 97 … is the first negative term?

Solution:
Here a = 105 and d = 101 – 105 = -4
Let an be the first negative term.
$\Rightarrow a_{2n} < 0$
$\Rightarrow a + (n - 1)d < 0$
$\Rightarrow 105 + (n - 1)(-4)$

Question 20.
How many three digit numbers are divisible by 7?

Solution:
The first three digit number which is divisible by 7 is 105 and the last digit which is divisible by 7 is 994.

This is an A.P. in which a = 105, d = 7 and $t_n = 994$.
We know that n^{th} term of A.P is given by
$t_n = a + (n - 1)d$.
$\Rightarrow 994 = 105 + (n - 1)7$
$\Rightarrow 889 = 7n - 7$
$\Rightarrow 896 = 7n$
$\Rightarrow n = 128$
∴ There are 128 three digit numbers which are divisible by 7.

Question 21.
Divide 216 into three parts which are in A.P. and the product of the two smaller parts is 5040.

Solution:
Let the three parts of 216 in A.P be $(a - d), a, (a + d)$.
$\Rightarrow a - d + a + a + d = 216$
$\Rightarrow 3a = 216$
$\Rightarrow a = 72$

Given that the product of the two smaller parts is 5040.
$\Rightarrow a(a - d) = 5040$
$\Rightarrow 72(72 - d) = 5040$
$\Rightarrow 72 - d = 70$
$\Rightarrow d = 2$
$\therefore a - d = 72 - 2 = 70$, $a = 72$ and $a + d = 72 + 2 = 74$
Therefore the three parts of 216 are 70, 72 and 74.

Question 22.
Can $2n^2 - 7$ be the n^{th} term of an A.P? Explain.

Solution:
We have $2n^2 - 7$,
Substitute $n = 1, 2, 3, \ldots$, we get
$2(1)^2 - 7, 2(2)^2 - 7, 2(3)^2 - 7, 2(4)^2 - 7, \ldots.$
-5, 1, 11, ….

Difference between the first and second term = $1 - (-5) = 6$
And Difference between the second and third term = $11 - 1 = 10$
Here, the common difference is not same.
Therefore the nth term of an A.P can't be $2n^2 - 7$.

Question 23.
Find the sum of the A.P., 14, 21, 28, …, 168.

Solution:
Here $a = 14$, $d = 7$ and $t_n = 168$
$t_n = a + (n - 1)d$
$\Rightarrow 168 = 14 + (n - 1)7$
$\Rightarrow 154 = 7n - 7$
$\Rightarrow 154 = 7n - 7$
$\Rightarrow 161 = 7n$
$\Rightarrow n = 23$
We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_{23} = \frac{23}{2}[2 \times 14 + (23 - 1)7]$$

$$= \frac{23}{2}(28 + 154)$$

$$= \frac{23}{2} \times 182$$

$$= 2093$$

Therefore the sum of the A.P., 14, 21, 28, …, 168 is 2093.

Question 24.
The first term of an A.P. is 20 and the sum of its first seven terms is 2100; find the 31^{st} term of this A.P.

Solution:
Here a = 20 and S_7 = 2100
We know that,

$$S_n = \frac{n}{2}[2a - (n - 1)d]$$

$$\Rightarrow S_7 = \frac{7}{2}[2 \times 20 - (7 - 1)d]$$

$$\Rightarrow 2100 = \frac{7}{2}(40 - 6d)$$

$$\Rightarrow 4200 = 7(40 - 6d)$$

$$\Rightarrow 600 = 40 - 6d$$

$$\Rightarrow d = \frac{560}{6}$$

To find: t_{31} =?
$t_n = a + (n - 1)d$

$$\Rightarrow t_{31} = 20 - (31 - 1)\frac{560}{6}$$

$$= 20 - 30 \times \frac{560}{6}$$

$$= 20 - 5 \times 560$$

$$= 2820$$

Therefore the 31st term of the given A.P. is 2820.

Question 25.
Find the sum of last 8 terms of the A.P. -12, -10, -8, ……, 58.

Solution:
First we will reverse the given A.P. as we have to find the sum of last 8 terms of the A.P. 58, …., -8, -10, -12.

Here a = 58 , d = -2

$$S_n = \frac{n}{2}[2a - (n - 1)d]$$
$$\Rightarrow S_8 = \frac{8}{2}[2 \times 58 - (8 - 1)(-2)]$$
$$= 4(116 - 14)$$
$$= 4 \times 102$$
$$= 408$$

Therefore the sum of last 8 terms of the A.P. -12, -10, -8, ……, 58 is 408.