2. Exponents (Powers)

Exercise 2 (A)

Question 1.

Evaluate:

(i) $(3^{-1} \times 9^{-1}) \div 3^{-2}$

(ii) $(3^{-1} \times 4^{-1}) \div 6^{-1}$

(iii) $(2^{-1} + 3^{-1})^3$

(iv) $(3^{-1} \div 4^{-1})^2$

(v) $(2^2 + 3^2) \times \left(\frac{1}{2}\right)^2$

(vi) $(5^2 - 3^2) \times \left(\frac{2}{3}\right)^{-3}$

(vii) $\left[\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right] \div \left(\frac{1}{6}\right)^{-3}$

(viii) $\left[\left(-\frac{3}{4}\right)^{-2}\right]^2$

(ix) $\left\{\left(\frac{3}{5}\right)^{-2}\right\}^{-2}$

(x) $(5^{-1} \times 3^{-1}) \div 6^{-1}$

Solution:

(*i*) $(3^{-1} \times 9^{-1}) \div 3^{-2}$

$$= \left(\frac{1}{3} \times \frac{1}{9}\right) \div \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{1}{27} \div \frac{1}{9}$$

$$= \frac{1}{27} \times \frac{9}{1} = \frac{1}{3}$$

(*ii*) $(3^{-1} \times 4^{-1}) \div 6^{-1}$

$$= \left(\frac{1}{3} \times \frac{1}{4}\right) \div \frac{1}{6}$$

$$= \frac{1}{12} \div \frac{1}{6}$$

$$= \frac{1}{12} \times \frac{6}{1} = \frac{1}{2}$$

(*iii*) $(2^{-1} + 3^{-1})^3$

$$= \left(\frac{1}{2} + \frac{1}{3}\right)^3 = \left(\frac{1 \times 3}{2 \times 3} + \frac{1 \times 2}{3 \times 2}\right)^3$$

$$= \left(\frac{3+2}{6}\right)^3 = \left(\frac{5}{6}\right)^3$$

$$= \frac{5 \times 5 \times 5}{6 \times 6 \times 6} = \frac{125}{216}$$

(*iv*) $(3^{-1} \div 4^{-1})^2$

$$= \left(\frac{1}{3} \div \frac{1}{4}\right)^2$$

$$= \left(\frac{1}{3}\times\frac{4}{1}\right)^2 = \left(\frac{4}{3}\right)^2$$

$$= \frac{16}{9} = 1\frac{7}{9}$$

(*v*) $(2^2 + 3^2) \times \left(\frac{1}{2}\right)^2$

$$= (2 \times 2) + (3 \times 3) \times \left(\frac{1}{2}\times\frac{1}{2}\right)$$

$$= 4 + 9 \times \frac{1}{4} = \frac{13}{4} = 3\frac{1}{4}$$

(*vi*) $(5^2 - 3^2) \times \left(\frac{2}{3}\right)^{-3}$

$$= (5 \times 5) - (3 \times 3) \times \left(\frac{3}{2}\right)^3$$

$$= 25 - 9 \times \left(\frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}\right)$$

$$= 16 \times \frac{27}{8} = 54$$

(*vii*) $\left[\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right] \div \left(\frac{1}{6}\right)^{-3}$

$$= \left[\left(\frac{4}{1}\right)^3 - \left(\frac{3}{1}\right)^3\right] \div \left(\frac{6}{1}\right)^3$$

$$= \left(\frac{4}{1}\times\frac{4}{1}\times\frac{4}{1}-\frac{3}{1}\times\frac{3}{1}\times\frac{3}{1}\right) \div \left(\frac{6}{1}\right)^3$$

$$= 64 - 27 \times \left(\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\right)$$

$$= 37 \times \frac{1}{216} = \frac{37}{216}$$

(viii) $\left[\left(-\frac{3}{4}\right)^{-2}\right]^2 = \left(-\frac{3}{4}\right)^{-2\times 2} = \left(-\frac{3}{4}\right)^{-4}$

$$= \left(\frac{4}{3}\right)^4 = \frac{4\times4\times4\times4}{3\times3\times3\times3}$$

$$= \frac{256}{81} = 3\frac{13}{81}$$

(ix) $\left\{\left(\frac{3}{5}\right)^{-2}\right\}^{-2} = \left(\frac{3}{5}\right)^{-2\times(-2)} = \left(\frac{3}{5}\right)^4$

$$= \frac{3\times3\times3\times3}{5\times5\times5\times5} = \frac{81}{625}$$

(x) $(5^{-1} \times 3^{-1}) \div 6^{-1}$

$$= \left(\frac{1}{5}\times\frac{1}{3}\right) \div \frac{1}{6}$$

$$= \frac{1}{15} \div \frac{1}{6}$$

$$= \frac{1}{15} \times \frac{6}{1} = \frac{2}{5}$$

Question 2.
If 1125 = 3^m x 5^n; find m and n.
Solution:

$$1125 = 3^2 \times 5^3$$

The factors of 1125 are 3 × 3 × 5 × 5 × 5

3	1125
3	375
5	125
5	25
5	5
	1

$\therefore$ $1125 = 3 \times 3 \times 5 \times 5 \times 5$

Now comparing, $3^2 \times 5^3 = 3^m \times 5^n$

$\therefore$ $m = 2$ $n = 3$

Question 3.
Find x, if $9 \times 3^x = (27)^{2x-3}$
Solution:

$$9 \times 3^x = (27)^{2x-3}$$
$$3^2 \times 3^x = (3 \times 3 \times 3)^{2x-3}$$
$$\Rightarrow 3^{x+2} = (3)^{3(2x-3)}$$
$$\Rightarrow 3^{x+2} = (3)^{6x-9}$$

Since, bases are same, compare them,

$$x + 2 = 6x - 9$$
$$6x - x = 9 + 2$$
$$\Rightarrow 5x = 11$$
$$\Rightarrow x = \frac{11}{2}$$
$$\Rightarrow x = 2\frac{1}{5}$$

Exercise 2 (B)

Question 1.

Compute:

(i) $1^8 \times 3^0 \times 5^3 \times 2^2$

(ii) $(4^7)^2 \times (4^{-3})^4$

(iii) $(2^{-9} \div 2^{-11})^3$

(iv) $\left(\frac{2}{3}\right)^{-4} \times \left(\frac{27}{8}\right)^{-2}$

(v) $\left(\frac{56}{28}\right)^0 \div \left(\frac{2}{5}\right)^3 \times \frac{16}{25}$

(vi) $(12)^{-2} \times 3^3$

(vii) $(-5)^4 \times (-5)^6 \div (-5)^9$

(viii) $\left(-\frac{1}{3}\right)^4 \div \left(-\frac{1}{3}\right)^8 \times \left(-\frac{1}{3}\right)^5$

(ix) $9^0 \times 4^{-1} \div 2^{-4}$

(x) $(625)^{-\frac{3}{4}}$

Solution:

(*xi*) $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

(*xii*) $\left(\frac{1}{32}\right)^{-\frac{2}{5}}$

(*xiii*) $(125)^{-\frac{2}{3}} \div (8)^{\frac{2}{3}}$

(*xiv*) $(243)^{\frac{2}{5}} \div (32)^{-\frac{2}{5}}$

(*xv*) $(-3)^4 - \left(\sqrt[4]{3}\right)^0 \times (-2)^5 \div (64)^{\frac{2}{3}}$

(*xvi*) $(27)^{\frac{2}{3}} \div \left(\frac{81}{16}\right)^{-\frac{1}{4}}$

(*i*) $1^8 \times 3^0 \times 5^3 \times 2^2$

$= 1 \times 1 \times 5 \times 5 \times 5 \times 2 \times 2$

$= 125 \times 4$

$= 500$

(*ii*) $(4^7)^2 \times (4^{-3})^4$

$= 4^{14} \times 4^{-12}$

$= 4^{14-12} = 4^2$

$= 4 \times 4$

$= 16$

(*iii*) $\left(2^{-9} \div 2^{-11}\right)^3 = \left(\frac{2^{-9}}{2^{-11}}\right)^3$

$= (2^{-9+11})^3$

$= (2^2)^3 = 2^6$

$$= 2\times2\times2\times2\times2\times2$$
$$= 64$$

(iv) $\left(\frac{2}{3}\right)^{-4}\times\left(\frac{27}{8}\right)^{-2} = \left(\frac{2}{3}\right)^{-4}\times\left(\frac{3^3}{2^3}\right)^{-2}$

$$= \frac{2^{-4}}{3^{-4}}\times\frac{3^{-6}}{2^{-6}} = \frac{2^{-4}}{2^{-6}}\times\frac{3^{-6}}{3^{-4}}$$

$$= 2^{-4+6}\times\frac{1}{3^{-4+6}} = \frac{2^2}{3^2}$$

$$= \frac{4}{9}$$

(v) $\left(\frac{56}{28}\right)^{0}\div\left(\frac{2}{5}\right)^{3}\times\frac{16}{25}$

$$= 1\div\frac{2^3}{5^3}\times\frac{2\times2\times2\times2}{5\times5}$$

$$\left[\because\left(\frac{56}{28}\right)^{0}=1\right]$$

$$= 1\times\frac{5^3}{2^3}\times\frac{2^4}{5^2} = 5^{3-2}\times2^{4-3}$$

$$= 5^1\times2^1 = 10$$

(vi) $(12)^{-2}\times3^3 = (2\times2\times3)^{-2}\times3^3$

$$= (2^2\times3)^{-2}\times3^3$$
$$= 2^{-2\times2}\times3^{-2}\times3^3$$
$$= 2^{-4}\times3^{-2+3}$$
$$= 2^{-4}\times3^{1}$$
$$= \frac{3}{2^4} = \frac{3}{2\times2\times2\times2}$$
$$= \frac{3}{16}$$

(vii) $(-5)^4\times(-5)^6\div(-5)^9$

$$= (-5)^4\times(-5)^6\times\frac{1}{(-5)^9}$$
$$= (-5)^{4+6-9}$$

$$= (-5)^{10-9} = -5$$

(*viii*) $\left(-\frac{1}{3}\right)^4 \div \left(-\frac{1}{3}\right)^8 \times \left(-\frac{1}{3}\right)^5$

$$= \left(-\frac{1}{3}\right)^4 \times \frac{1}{\left(-\frac{1}{3}\right)^8} \times \left(-\frac{1}{3}\right)^5$$

$$= \left(-\frac{1}{3}\right)^{4+5-8} = \left(-\frac{1}{3}\right)^{9-8}$$

$$= -\frac{1}{3}$$

(*ix*) $9^0 \times 4^{-1} \div 2^{-4} = 1 \times \frac{1}{4^1} \times \frac{1}{2^{-4}}$

$$= 1 \times \frac{1}{4} \times 2^4 = 1 \times \frac{1}{2^2} \times 2^4$$

$$= 2^{4-2} = 2^2 = 4$$

(*x*) $(625)^{-\frac{3}{4}} = (5 \times 5 \times 5 \times 5)^{-\frac{3}{4}}$

$$= \left(5^4\right)^{-\frac{3}{4}} = 5^{4 \times -\frac{3}{4}}$$

$$= 5^{-3} = \frac{1}{5^3}$$

$$= \frac{1}{5 \times 5 \times 5}$$

$$= \frac{1}{125}$$

(*xi*) $\left(\frac{27}{64}\right)^{-\frac{2}{3}} = \left[\frac{(3^3)}{(4^3)}\right]^{-\frac{2}{3}}$

$$= \frac{3^{3\times-\frac{2}{3}}}{4^{3\times\frac{-2}{3}}} = \frac{3^{-2}}{4^{-2}}$$

$$= \frac{4^2}{3^2} = \frac{4\times4}{3\times3} = \frac{16}{9}$$

$$= 1\frac{7}{9}$$

(*xii*) $\left(\frac{1}{32}\right)^{-\frac{2}{5}} = \left(\frac{1}{2\times2\times2\times2\times2}\right)^{\frac{2}{5}}$

$$= \left(\frac{1}{2^5}\right)^{-\frac{2}{5}} = \frac{1}{2^{5\times-\frac{2}{5}}}$$

$$= \frac{1}{2^{-2}} = 2^2 = 4$$

(*xiii*) $(125)^{-\frac{2}{3}} \div (8)^{\frac{2}{3}} = \left(5^3\right)^{-\frac{2}{3}} \div \left(2^3\right)^{\frac{2}{3}}$

$$= 5^{-\frac{2}{3}\times3} \div 2^{3\times\frac{2}{3}}$$

$$= 5^{-2} \div 2^2 = \frac{1}{5^2}\times\frac{1}{2^2}$$

$$= \frac{1}{25}\times\frac{1}{4} = \frac{1}{100}$$

(*xiv*) $(243)^{\frac{2}{5}} \div (32)^{-\frac{2}{5}}$

$$= (3\times3\times3\times3\times3)^{\frac{2}{5}} \div (2\times2\times2\times2\times2)^{-\frac{2}{5}}$$

$$= \left(3^5\right)^{\frac{2}{5}} \div \left(2^5\right)^{-\frac{2}{5}}$$

$$= 3^{5\times\frac{2}{5}} \div 2^{-\frac{2}{5}\times5} = 3^2 \div 2^{-2}$$

$$= 3^2 \times \frac{1}{2^{-2}} = 3^2\times2^{+2}$$

$$= 3\times3\times2\times2 = 36$$

(*xv*) $(-3)^4 - \left(\sqrt[4]{3}\right)^0 \times (-2)^5 \div (64)^{\frac{2}{3}}$

$$= (-3\times-3\times-3\times-3)$$

$$- \;1\times-2\times-2\times-2\times-2\times-2 \div \left(2^6\right)^{\frac{2}{3}}$$

Note : $\left(\sqrt[4]{3}\right)^0 = 1$

$$= 3^4 + 2^5 \div 2^{6\times\frac{2}{3}}$$

$$= 3^4 + 2^5 \div 2^4 = 3^4 + \frac{2^5}{2^4}$$

$$= 3^4 + 2^{5-4} = 3^4 + 2 = 3\times3\times3\times3+2$$

$$= 81+2 = 83$$

(*xvi*) $(27)^{\frac{2}{3}} \div \left(\frac{81}{16}\right)^{-\frac{1}{4}} = \left(3^3\right)^{\frac{2}{3}} \div \left(\frac{3^4}{2^4}\right)^{-\frac{1}{4}}$

$$= 3^{3\times\frac{2}{3}} \div \frac{3^{-\frac{1}{4}\times4}}{2^{-\frac{1}{4}\times4}} = 3^2 \div \frac{3^{-1}}{2^{-1}}$$

$$= 3^2 \times \frac{2^{-1}}{3^{-1}}$$

$$= 3^{2+1}\times2^{-1} = 3^3\times\frac{1}{2^{+1}}$$

$$= \frac{3\times3\times3}{2} = \frac{27}{2} = 13\frac{1}{2}$$

Question 2.
Simplify:

(i) $8^{\frac{4}{3}} + 25^{\frac{3}{2}} - \left(\frac{1}{27}\right)^{-\frac{2}{3}}$

(ii) $[(64)^{-2}]^{-3} \div [\{(-8)^2\}^3]^2$

(iii) $(2^{-3} - 2^{-4})(2^{-3} + 2^{-4})$

Solution:

(i) $8^{\frac{4}{3}} + 25^{\frac{3}{2}} - \left(\frac{1}{27}\right)^{-\frac{2}{3}}$

$= (2^3)^{\frac{4}{3}} + (5^2)^{\frac{3}{2}} - \left(\frac{1}{3^3}\right)^{-\frac{2}{3}}$

$= 2^{3\times\frac{4}{3}} + 5^{2\times\frac{3}{2}} - \frac{1}{3^{3\times\left(\frac{-2}{3}\right)}}$

$= 2^4 + 5^3 - \frac{1}{3^{-2}}$

$= 16 + 125 - 3^2$

$= 141 - 9 = 132$

$$= 2^{3\times\frac{4}{3}} + 5^{2\times\frac{3}{2}} - \frac{1}{3^{3\times\left(\frac{-2}{3}\right)}}$$

$$= 2^4 + 5^3 - \frac{1}{3^{-2}}$$

$= 16 + 125 - 3^2$

$= 141 - 9 = 132$

(*ii*) $[(64)^{-2}]^{-3} \div [\{(-8)^2\}^3]^2$

$= (2^6)^{-2\times-3} \div (-8)^{2\times3\times2}$

$= 2^{6\times(6)} \div (-8)^{12}$

$= 2^{+36} \div (-8)^{12}$

$= 2^{+36} \div [(-2)^3]^{12} = 2^{36} \div (-2)^{36}$

$$= \frac{2^{36}}{(-2)^{36}} = \frac{2^{36}}{2^{36}} \qquad (\because 36 \text{ is even})$$

$= 2^{36-36} = 2^0 = 1 \qquad (\because a^0 = 1)$

(*iii*) $(2^{-3} - 2^{-4})(2^{-3} + 2^{-4})$

$= (2^{-3})^2 - (2^{-4})^2$

$\{\because (a - b)(a + b) = a^2 - b^2\}$

$$= 2^{-6} - 2^{-8} = \frac{1}{2^6} - \frac{1}{2^8}$$

$$= \frac{1}{64} - \frac{1}{256}$$

$$= \frac{4-1}{256} = \frac{3}{256}$$

Question 3.
Evaluate:

(i) $(-5)^0$

(ii) $8^0 + 4^0 + 2^0$

(iii) $(8 + 4 + 2)^0$

(iv) $4x^0$

(v) $(4x)^0$

(vi) $[(10^3)^0]^5$

(vii) $(7x^0)^2$

(viii) $9^0 + 9^{-1} - 9^{-2} + 9^{\frac{1}{2}} - 9^{-\frac{1}{2}}$

Solution:

(i) $(-5)^0 = 1$ $\quad (\because a^0 = 1)$

(ii) $8^0 + 4^0 + 2^0$

$= 1 + 1 + 1 = 3$ $\quad (\because a^0 = 1)$

(iii) $(8 + 4 + 2)^0 = (14)^0 = 1$ $\quad (\because a^0 = 1)$

(iv) $4x^0 = 4 \times 1 = 4$

(v) $(4x)^0 = 1$

(vi) $[(10^3)^0]^5 = 10^{3 \times 0 \times 5} = 10^0 = 1$

(vii) $(7x^0)^2 = 7^2 \times x^{0 \times 2} = 49 \times 1 = 49$

(viii) $9^0 + 9^{-1} - 9^{-2} + 9^{\frac{1}{2}} - 9^{-\frac{1}{2}}$

$$= 1 + \frac{1}{9} - \frac{1}{9^2} + (3^2)^{\frac{1}{2}} - (3^2)^{-\frac{1}{2}}$$

$$= 1 + \frac{1}{9} - \frac{1}{81} + 3^{2\times\frac{1}{2}} - 3^{2\times\left(-\frac{1}{2}\right)}$$

$$= 1 + \frac{1}{9} - \frac{1}{81} + 3 - 3^{-1}$$

$$= 1 + \frac{1}{9} - \frac{1}{81} + \frac{3}{1} - \frac{1}{3}$$

$$= \frac{81+9-1+243-27}{81} = \frac{333-28}{81}$$

$$= \frac{305}{81} = 3\frac{62}{81}$$

Question 4.
Simplify:

(i) $\frac{a^5b^2}{a^2b^{-3}}$

(ii) $15y^8 \div 3y^3$

(iii) $x^{10}y^6 \div x^3y^{-2}$

(iv) $5z^{16} \div 15z^{-11}$

(v) $(36x^2)^{\frac{1}{2}}$

(vi) $(125x^{-3})^{\frac{1}{3}}$

(vii) $(2x^2y^{-3})^{-2}$

(viii) $(27x^{-3}y^6)^{\frac{2}{3}}$

(ix) $(-2x^{2/3}y^{-3/2})^6$

Solution:

(i) $\dfrac{a^5b^2}{a^2b^{-3}} = a^{5-2}.b^{2+3}$

$= a^3b^5$

(ii) $15y^8 \div 3y^3 = \dfrac{15y^8}{3y^3}$

$= 5y^{8-3}$

$= 5y^5$

(iii) $x^{10}y^6 \div x^3y^{-2} = \dfrac{x^{10}y^6}{x^3y^{-2}}$

$= x^{10-3}.y^{6+2}$

$= x^7y^8$

(iv) $5z^{16} \div 15z^{-11} = \dfrac{5z^{16}}{15z^{-11}}$

$= \dfrac{5}{15}z^{16+11}$

$= \dfrac{1}{3}z^{27}$

(v) $(36x^2)^{1/2} = (36)^{1/2}.\,x^{2\times\frac{1}{2}}$

$= (6\times6)^{1/2}.x = (6^2)^{1/2}.x$

$= 6x$

(vi) $(125x^{-3})^{1/3} = (125)^{1/3}.x^{-3\times1/3}$

$= (5\times5\times5)^{1/3}.x^{-1}$

$$(5^3)^{\frac{1}{3}}.x^{-1} = 5x^{-1}$$

$$= \frac{5}{x} = 5x^{-1}$$

(vii) $(2x^2y^{-3})^{-2} = 2^{-2}.x^{2\times-2}.y^{-3\times-2}$

$$= \frac{1}{2^2}x^{-4}.y^6$$

$$= \frac{1}{4}\times\frac{y^6}{x^4}$$

$$= \frac{y^6}{4x^4} = \frac{1}{4}.y^6.x^{-4}$$

(viii) $(27x^{-3}y^6)^{2/3} = (27)^{2/3}.x^{-3\times\frac{2}{3}}.y^{6\times\frac{2}{3}}$

$$= (3\times3\times3)^{2/3}.x^{-2}.y^4$$

$$= [(3\times3\times3)^{1/3}]^2.x^{-2}.y^4$$

$$= 3^2.x^{-2}y^4$$

$$= 9x^{-2}y^4$$

$$= \frac{9y^4}{x^2} = 9x^{-2}y^4$$

(ix) $\left(-2x^{2/3}y^{-3/2}\right)^6$

$$= (-2)^6.x^{2/3\times6}.y^{-3/2\times6}$$

$$= 64x^4y^{-9}$$

$$= \frac{64x^4}{y^9}$$

$$= 64x^4y^{-9}$$

Question 5.
Simplify:

$(x^{a+b})^{a-b}.(x^{b+c})^{b-c}.(x^{c+a})^{c-a}$

Solution:

$$(x^{a+b})^{a-b}.(x^{b+c})^{b-c}.(x^{c+a})^{c-a}$$

$$= x^{(a+b)(a-b)}.x^{(b+c)(b-c)}.x^{(c+a)(c-a)}$$

$$= x^{a^2-b^2}.x^{b^2-c^2}.x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= x^0$$

$$= 1$$

Question 6.

Simplify:

(i) $\sqrt[5]{x^{20}y^{-10}z^5} + \frac{x^3}{y^3}$

Solution:

(ii) $\left(\frac{256a^{16}}{81b^4}\right)^{\frac{-3}{4}}$

(i) $\sqrt[5]{x^{20}y^{-10}z^{5}} \div \frac{x^3}{y^3}$

$= (x^{20}y^{-10}z^{5})^{1/5} \div \frac{x^3}{y^3}$

$= x^{20\times\frac{1}{5}} . y^{-10\times\frac{1}{5}} . z^{5\times\frac{1}{5}} \div \frac{x^3}{y^3}$

$= x^4 . y^{-2} . z^1 \times \frac{y^3}{x^3}$

$= x^{4-3} . y^{-2+3} . z^1$

$= xyz$

(ii) $\left[\frac{256a^{16}}{81b^4}\right]^{-3/4} = \left[\frac{4^4 a^{16}}{3^4 b^4}\right]^{\frac{-3}{4}}$

256	=	4×4×4×4	=	4^4
81	=	3×3×3×3	=	3^4

$= \frac{4^{4\times\frac{-3}{4}} . a^{16\times\frac{-3}{4}}}{3^{4\times\frac{-3}{4}} . b^{4\times\frac{-3}{4}}}$

$= \frac{4^{-3} . a^{-12}}{3^{-3} . b^{-3}}$

$= \frac{3^3 b^3}{4^3 a^{12}}$

$= \frac{27b^3}{64a^{12}}$

$= \frac{27}{64} . a^{-12}b^3$

Note :

$4^{-3} = \frac{1}{4^3}$

$\frac{1}{3^{-3}} = 3^3$

$a^{-12} = \frac{1}{a^{12}}$

$\frac{1}{b^{-3}} = b^3$

Question 7.

(i) $(a^{-2})^{-2} . (ab)^{-3}$

(ii) $(x^n y^{-m})^4 \times (x^3 y^{-2})^{-n}$

(iii) $\left(\dfrac{125a^{-3}}{y^6}\right)^{\frac{-1}{3}}$

(iv) $\left(\dfrac{32x^{-5}}{243y^{-5}}\right)^{\frac{-1}{5}}$

(v) $(a^{-2}b)^{1/2} \times (ab^{-3})^{1/3}$

(vi) $(xy)^{m-n}.\ (yz)^{n-l}.(zx)^{l-m}$

Solution:

(i) $(a^{-2}b)^{-2}.\ (ab)^{-3}$

$= (a^{-2\times-2}.b^{-2})\ .\ (a^{-3}.b^{-3})$

$= a^{+4}.b^{-2}.a^{-3}.b^{-3}$

$= a^{4-3}.\ b^{-2-3}$

$= ab^{-5}$

$= \dfrac{a}{b^5}$ **Ans.**

(ii) $(x^n y^{-m})^4 \times (x^3 y^{-2})^{-n}$

$= x^{4n}\ y^{-4m} \times x^{-3n} y^{2n}$

$= x^{4n-3n}.y^{-4m+2n}$

$= x^n y^{-4m+2n}$

(iii) $\left[\dfrac{125a^{-3}}{y^6}\right]^{-1/3} = \left[\dfrac{5^3 a^{-3}}{y^6}\right]^{-1/3}$

$125 = 5 \times 5 \times 5 = 5^3$

$= \dfrac{5^{3\times\frac{-1}{3}}.a^{-3\times\frac{-1}{3}}}{y^{6\times\frac{-1}{3}}}$

$= \dfrac{5^{-1}.a^{1}}{y^{-2}}$

(iv) $\left[\frac{32x^{-5}}{243y^{-5}}\right]^{\frac{-1}{5}} = \left[\frac{2^5x^{-5}}{3^5y^{-5}}\right]^{\frac{-1}{5}}$

$32=2\times2\times2\times2\times2=2^5$
$243=3\times3\times3\times3\times3=3^5$

$$= \frac{2^{5\times\frac{-1}{5}}.x^{-5\times\frac{-1}{5}}}{3^{5\times\frac{-1}{5}}y^{-5\times\frac{-1}{5}}}$$

$$= \frac{2^{-1}x^{+1}}{3^{-1}y^{+1}}$$

$$= \frac{3x}{2y}$$

(v) $(a^{-2}b)^{\frac{1}{2}}\times(ab^{-3})^{\frac{1}{3}}$

$$= (a^{-2\times\frac{1}{2}}.b^{1/2})\times(a^{1/3}b^{-3\times\frac{1}{3}})$$

$$= a^{-1}b^{1/2}\times a^{1/3}b^{-1}$$

$$= a^{-1+\frac{1}{3}}\ b^{\frac{1}{2}-1}$$

$$= a^{-2/3}b^{-1/2}$$

$$= \frac{1}{a^{2/3}b^{1/2}}$$

(vi) $(xy)^{m-n}.(yz)^{n-l}(xz)^{l-m}$

$$= x^{m-n}.\ y^{m-n}.y^{n-l}.z^{n-l}.x^{l-m}.z^{l-m}$$

$$= x^{m-n+l-m}.y^{m-n+n-l}.z^{n-l+l-m}$$

$$= x^{l-n}.y^{m-l}.z^{n-m}$$

Question 8.
Show that:

$$\left(\frac{x^a}{x^{-b}}\right)^{a-b} \cdot \left(\frac{x^b}{x^{-c}}\right)^{b-c} \cdot \left(\frac{x^c}{x^{-a}}\right)^{c-a} = 1$$

Solution:

L.H.S.

$$\left(\frac{x^a}{x^{-b}}\right)^{a-b} \cdot \left(\frac{x^b}{x^{-c}}\right)^{b-c} \cdot \left(\frac{x^c}{x^{-a}}\right)^{c-a}$$

$$= (x^{a+b})^{a-b}, (x^{b+c})^{b-c}.(x^{c+a})^{c-a}$$

$$= x^{(a+b)(a-b)}.x^{(b+c)(b-c)}.x^{(c+a)(c-a)}$$

$$= x^{a^2-b^2}.x^{b^2-c^2}.x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= x^0$$

$$= 1 = \textbf{R.H.S.}$$

Question 9.

Evaluate:

$$\frac{x^{5+n}(x^2)^{3n+1}}{x^{7n-2}}$$

Solution:

$$\frac{x^{5+n} \times (x^2)^{3n+1}}{x^{7n-2}}$$

$$= \frac{x^{5+n} \times x^{2(3n+1)}}{x^{7n-2}}$$

$$= \frac{x^{5+n} \times x^{6n+2}}{x^{7n-2}}$$

$$= x^{5+n+6n+2-7n+2}$$

$$= x^9$$

Question 10.
Evaluate:

$$\frac{a^{2n+1}\times a^{(2n+1)(2n-1)}}{a^{n(4n-1)}\times(a^2)^{2n+3}}$$

Solution:

$$\frac{a^{2n+1}\times a^{(2n+1)(2n-1)}}{a^{n(4n-1)}\times(a^2)^{2n+3}}$$

$$=\frac{a^{2n+1}\times a^{(2n)^2-(1)^2}}{a^{4n^2-n}\times a^{2(2n+3)}}$$

$$=\frac{a^{2n+1}\times a^{4n^2-1}}{a^{4n^2-n}\times a^{4n+6}}$$

$$= a^{2n+1+4n^2-1-4n^2+n-4n-6}$$

$$= a^{-n-6}$$

$$= a^{-(n+6)}$$

$$=\frac{1}{a^{n+6}}$$

Question 11.
$(m+n)^{-1}(m^{-1}+n^{-1}) = (mn)^{-1}$
Solution:

L.H.S. $(m+n)^{-1}(m^{-1}+n^{-1})$

$$=\frac{1}{m+n}\left(\frac{1}{m}+\frac{1}{n}\right)=\frac{1}{m+n}.\frac{n+m}{mn}=\frac{1}{mn}$$

$= (mn)^{-1}$

= R.H.S.

Hence proved.

Question 12.

Prove that:

$$(i)\ \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}=1$$

$$(ii)\ \frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}}=1$$

Solution:

$$(i)\ \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}=1$$

$$\text{L.H.S.}=\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}$$

$$\left(x^{a-b}\right)^{\frac{1}{ab}}\left(x^{b-c}\right)^{\frac{1}{bc}}\left(x^{c-a}\right)^{\frac{1}{ca}}$$

$$= x^{\frac{a-b}{ab}} \cdot x^{\frac{b-c}{bc}} \cdot x^{\frac{c-a}{ca}} \qquad \left\{\left(x^a\right)^b = x^{ab}\right\}$$

$$= x^{\frac{a-b}{ab}+\frac{b-c}{bc}+\frac{c-a}{ca}}$$

$$= x^{\frac{ac-bc+ab-ac+bc-ab}{abc}}$$

$$= x^0 = 1 = \text{R.H.S.} \qquad (\because x^0 = 1)$$

(ii) $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$

$$\text{L.H.S.} = \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$$

$$= \frac{1}{x^{a-a}+x^{a-b}} + \frac{1}{x^{b-b}+x^{b-a}}$$

$$= \frac{1}{x^a.x^{-a}+x^a.x^{-b}} + \frac{1}{x^b.x^{-b}+x^b.x^{-a}}$$

$$= \frac{1}{x^a\left(x^{-a}+x^{-b}\right)} + \frac{1}{x^b\left(x^{-b}+x^{-a}\right)}$$

$$= \frac{1}{\left(x^{-a}+x^{-b}\right)}\left[\frac{1}{x^a}+\frac{1}{x^b}\right]$$

$$= \frac{1}{x^{-a}+x^{-b}}\left[x^{-a}+x^{-b}\right] = 1 = \text{R.H.S.}$$

Question 13.

Find the values of n, when:

(i) $12^{-5} \times 12^{2n+1} = 12^{13} \div 12^7$

(ii) $\frac{a^{2n-3} \times (a^2)^{n+1}}{(a^4)^{-3}} = (a^3)^3 \div (a^6)^{-3}$

Solution:

(i) $12^{-5} \times 12^{2n+1} = 12^{13} \div 12^{7}$

$$12^{-5+2n+1} = \frac{12^{13}}{12^{7}}$$

$12^{2n-4} = 12^{13-7}$

$12^{2n-4} = 12^{6}$

Comparing both sides, we get

$2n - 4 = 6$

$\Rightarrow 2n = 6 + 4$

$\Rightarrow 2n = 10$

$\Rightarrow n = 5$

(ii) $$\frac{a^{2n-3} \times (a^2)^{n+1}}{(a^4)^{-3}} = (a^3)^3 \div (a^6)^{-3}$$

$$\frac{a^{2n-3} \times 2^{2n+2}}{a^{-12}} = a^9 \div a^{-18}$$

$$\frac{a^{2n-3} \times 2^{2n+2}}{a^{-12}} = \frac{a^9}{a^{-18}}$$

$a^{2n-3+2n+2-(-12)} = a^{9-(-18)}$

$a^{4n+11} = a^{27}$

Comparing both sides, we get

$4n + 11 = 27$

$\Rightarrow 4n = 27 - 11$

$$\Rightarrow n = \frac{16}{4} = 4$$

Question 14.

Simplify:

(i) $$\frac{a^{2n-3} \cdot a^{(2n+1)(n+2)}}{(a^3)^{2n+1} \cdot a^{n(2n+1)}}$$

(ii) $$\frac{x^{2n+7} \cdot (x^2)^{3n+2}}{x^{4(2n+3)}}$$

Solution:

(i) $\frac{a^{2n-3} \cdot a^{(2n+1)(n+2)}}{(a^3)^{2n+1} \cdot a^{n(2n+1)}}$

Given expression $= \frac{a^{2n+3} \cdot a^{(2n^2+4n+n+2)}}{a^{6n+3} \cdot a^{2n^2+n}}$

$= \frac{a^{2n+3+2n^2+5n+2}}{a^{6n+3+2n^2+n}} = \frac{a^{2n^2+7n+5}}{a^{2n^2+7n+3}}$

$= \frac{a^{(2n^2+7n+3)+2}}{a^{2n^2+7n+3}} = a^2$

(ii) $\frac{x^{2n+7} \cdot (x^2)^{3n+2}}{x^{4(2n+3)}}$

Given expression $= \frac{x^{2n+7} \cdot x^{6n+4}}{x^{8n+12}}$

$= \frac{x^{2n+7+6n+4}}{x^{8n+12}} = \frac{x^{8n++11}}{x^{8n+12}}$

$= x^{8n + 11 - 8n - 12} = x^{-1}$

$= \frac{1}{x}$