Chapter 7. Indices (Exponents)

Exercise 7(A)

Solution 1:

(i)

$$3^3 \times (243)^{-\frac{2}{3}} \times 9^{-\frac{1}{3}} = 3^3 \times (3 \times 3 \times 3 \times 3 \times 3)^{-\frac{2}{3}} \times (3 \times 3)^{-\frac{1}{3}}$$

$$= 3^3 \times \left(3^5\right)^{-\frac{2}{3}} \times \left(3^2\right)^{-\frac{1}{3}}$$

$$= 3^3 \times 3^{\left(-\frac{10}{3}\right)} \times 3^{-\frac{2}{3}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 3^{3-\frac{10}{3}-\frac{2}{3}} \qquad \left[a^m \times a^n \times a^p = a^{m+n+p}\right]$$

$$= 3^{\frac{9-10-2}{3}}$$

$$= 3^{\frac{9-12}{3}}$$

$$= 3^{-\frac{3}{3}}$$

$$= 3^{-1}$$

$$= \frac{1}{3}$$

(ii)

$$5^{-4} \times (125)^{\frac{5}{3}} \div (25)^{-\frac{1}{2}} = 5^{-4} \times (5 \times 5 \times 5)^{\frac{5}{3}} \div (5 \times 5)^{-\frac{1}{2}}$$

$$= 5^{-4} \times \left(5^3\right)^{\frac{5}{3}} \div \left(5^2\right)^{-\frac{1}{2}}$$

$$= 5^{-4} \times \left(5^{3 \times \frac{5}{3}}\right) \div \left(5^{2 \times \left(-\frac{1}{2}\right)}\right)$$

$$= \frac{5^{-4} \times 5^5}{5^{-1}}$$

$$= \frac{5^{5-4}}{5^{-1}}$$

$$= \frac{5^1}{5^{-1}}$$

$$= 5^{1-(-1)}$$

$$= 5^2$$

$$= 5 \times 5$$

$$= 25$$

(iii)

$$\left(\frac{27}{125}\right)^{\frac{2}{3}} \times \left(\frac{9}{25}\right)^{-\frac{3}{2}} = \left(\frac{3\times3\times3}{5\times5\times5}\right)^{\frac{2}{3}} \times \left(\frac{3\times3}{5\times5}\right)^{-\frac{3}{2}}$$

$$= \left[\left(\frac{3}{5}\right)^{3}\right]^{\frac{2}{3}} \times \left[\left(\frac{3}{5}\right)^{2}\right]^{-\frac{3}{2}}$$

$$= \left(\frac{3}{5}\right)^{3\times\frac{2}{3}} \times \left(\frac{3}{5}\right)^{2\times\left(-\frac{3}{2}\right)}$$

$$= \left(\frac{3}{5}\right)^{2} \times \left(\frac{3}{5}\right)^{-3}$$

$$= \left(\frac{3}{5}\right)^{2-3}$$

$$= \left(\frac{3}{5}\right)^{-1}$$

$$= \frac{1}{\frac{3}{5}}$$

$$= \frac{5}{3}$$

(iv)

$$7^0 \times (25)^{-\frac{3}{2}} - 5^{-3} = 7^0 \times (5\times5)^{-\frac{3}{2}} - 5^{-3}$$

$$= 7^0 \times \left(5^2\right)^{-\frac{3}{2}} - \frac{1}{5^3}$$

$$= 7^0 \times 5^{2\times\left(-\frac{3}{2}\right)} - \frac{1}{5^3}$$

$$= 7^0 \times 5^{-3} - \frac{1}{5^3}$$

$$= 1 \times 5^{-3} - \frac{1}{5^3}$$

$$= \frac{1}{5^3} - \frac{1}{5^3}$$

$$= \frac{1-1}{5\times5\times5}$$

$$= \frac{0}{125}$$

$$= 0$$

(v)

$$\left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \left(\frac{49}{9}\right)^{\frac{3}{2}} \div \left(\frac{343}{216}\right)^{\frac{2}{3}}$$

$$= \left(\frac{2\times2\times2\times2}{3\times3\times3\times3}\right)^{-\frac{3}{4}} \times \left(\frac{7\times7}{3\times3}\right)^{\frac{3}{2}} \div \left(\frac{7\times7\times7}{6\times6\times6}\right)^{\frac{2}{3}}$$

$$= \left[\left(\frac{2}{3}\right)^{4}\right]^{-\frac{3}{4}} \times \left[\left(\frac{7}{3}\right)^{2}\right]^{\frac{3}{2}} \div \left[\left(\frac{7}{6}\right)^{3}\right]^{\frac{2}{3}}$$

$$= \left(\frac{2}{3}\right)^{4\times\left(-\frac{3}{4}\right)} \times \left(\frac{7}{3}\right)^{2\times\frac{3}{2}} \div \left(\frac{7}{6}\right)^{3\times\frac{2}{3}}$$

$$= \left(\frac{2}{3}\right)^{-3} \times \left(\frac{7}{3}\right)^{3} \div \left(\frac{7}{6}\right)^{2}$$

$$= \frac{1}{\left(\frac{2}{3}\right)^{3}} \times \left(\frac{7}{3}\right)^{3} \times \frac{1}{\left(\frac{7}{6}\right)^{2}}$$

$$= \frac{1}{\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}} \times \frac{7}{3} \times \frac{7}{3} \times \frac{7}{3} \times \frac{1}{\frac{7}{6}\times\frac{7}{6}}$$

$$= \frac{1\times3\times3\times3}{2\times2\times2} \times \frac{7}{3} \times \frac{7}{3} \times \frac{7}{3} \times \frac{1\times6\times6}{7\times7}$$

$$= \frac{7\times3\times3}{2}$$

$$= \frac{63}{2}$$

$$= 31.5$$

Solution 2:

(i)

$$\left(8x^3 \div 125y^3\right)^{\frac{2}{3}} = \left(\frac{8x^3}{125y^3}\right)^{\frac{2}{3}}$$

$$= \left(\frac{2x\times2x\times2x}{5y\times5y\times5y}\right)^{\frac{2}{3}}$$

$$= \left[\left(\frac{2x}{5y}\right)^{3}\right]^{\frac{2}{3}}$$

$$= \left(\frac{2x}{5y}\right)^{3\times\frac{2}{3}}$$

$$= \left(\frac{2x}{5y}\right)^{2}$$

$$= \frac{2x}{5y} \times \frac{2x}{5y}$$

$$= \frac{4x^2}{25y^2}$$

(ii)

$$\left(a+b\right)^{-1}.\left(a^{-1}+b^{-1}\right)=\frac{1}{\left(a+b\right)}\times\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$=\frac{1}{\left(a+b\right)}\times\left(\frac{b+a}{ab}\right)$$

$$=\frac{1}{\left(a+b\right)}\times\frac{\left(a+b\right)}{ab}$$

$$=\frac{1}{ab}$$

(iii)

$$\frac{5^{n+3}-6\times5^{n+1}}{9\times5^{n}-5^{n}\times2^{2}}=\frac{5^{n+1}\times5^{2}-6\times5^{n+1}}{9\times5^{n}-5^{n}\times2^{2}}$$

$$=\frac{5^{n+1}\times\left(5^{2}-6\right)}{5^{n}\times\left(9-4\right)}$$

$$=\frac{5^{n}\times5^{1}\times\left(25-6\right)}{5^{n}\times\left(9-4\right)}$$

$$=\frac{5^{1}\times19}{5}$$

$$=19$$

(iv)

$$\left(3x^{2}\right)^{-3}\times\left(x^{9}\right)^{\frac{2}{3}}=\frac{1}{\left(3x^{2}\right)^{3}}\times x^{9\times\frac{2}{3}}$$

$$=\frac{1}{3^{3}x^{2\times3}}\times x^{6}$$

$$=\frac{1}{27x^{6}}\times x^{6}$$

$$=\frac{1}{27}$$

Solution 3:

(i)

$$\sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} = \sqrt{\frac{1}{2} \times \frac{1}{2}} + (0.1 \times 0.1)^{-\frac{1}{2}} - (3 \times 3 \times 3)^{\frac{2}{3}}$$

$$= \frac{1}{2} + \left[(0.1)^2\right]^{-\frac{1}{2}} - \left(3^2\right)^{\frac{2}{3}}$$

$$= \frac{1}{2} + (0.1)^{2 \times \left(-\frac{1}{2}\right)} - 3^{3 \times \frac{2}{3}}$$

$$= \frac{1}{2} + (0.1)^{-1} - 3^2$$

$$= \frac{1}{2} + \frac{1}{0.1} - 9$$

$$= \frac{1}{2} + \frac{10}{1} - 9$$

$$= \frac{1 + 20 - 18}{2}$$

$$= \frac{3}{2}$$

(ii)

$$\left(\frac{27}{8}\right)^{\frac{2}{3}} - \left(\frac{1}{4}\right)^{-2} + 5^0 = \left(\frac{3 \times 3 \times 3}{2 \times 2 \times 2}\right)^{\frac{2}{3}} - \left(\frac{1 \times 1}{2 \times 2}\right)^{-2} + 5^0$$

$$= \left[\left(\frac{3}{2}\right)^3\right]^{\frac{2}{3}} - \left[\left(\frac{1}{2}\right)^2\right]^{-2} + 1$$

$$= \left(\frac{3}{2}\right)^{3 \times \frac{2}{3}} - \left(\frac{1}{2}\right)^{2 \times (-2)} + 1$$

$$= \left(\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^{-4} + 1$$

$$= \frac{3}{2} \times \frac{3}{2} - \frac{1}{\left(\frac{1}{2}\right)^4} + 1$$

$$= \frac{9}{4} - \frac{1}{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}} + 1$$

$$= \frac{9}{4} - \frac{1}{\frac{1}{16}} + 1$$

$$= \frac{9}{4} - 16 + 1$$

$$= \frac{9 - 64 + 4}{4}$$

$$= \frac{-51}{4}$$

Solution 4:

(i)

$$\left(\frac{3^{-4}}{2^{-8}}\right)^{\frac{1}{4}} = \left(\frac{2^8}{3^4}\right)^{\frac{1}{4}}$$

$$= \frac{\left(2^8\right)^{\frac{1}{4}}}{\left(3^4\right)^{\frac{1}{4}}}$$

$$= \frac{2^{8\times\frac{1}{4}}}{3^{4\times\frac{1}{4}}}$$

$$= \frac{2^2}{3}$$

$$= \frac{4}{3}$$

(ii)

$$\left(\frac{27^{-3}}{9^{-3}}\right)^{\frac{1}{5}} = \left(\frac{9^3}{27^3}\right)^{\frac{1}{5}}$$

$$= \left(\frac{\left(3^2\right)^3}{\left(3^3\right)^3}\right)^{\frac{1}{5}}$$

$$= \left[\left(\frac{3^2}{3^3}\right)^3\right]^{\frac{1}{5}}$$

$$= \left[\left(\frac{1}{3}\right)^3\right]^{\frac{1}{5}}$$

$$= \left(\frac{1}{3}\right)^{3\times\frac{1}{5}}$$

$$= \frac{1}{3^{\frac{3}{5}}}$$

(iii)

$$(32)^{-\frac{2}{5}} \div (125)^{-\frac{2}{3}} = \frac{(32)^{-\frac{2}{5}}}{(125)^{-\frac{2}{3}}}$$

$$= \frac{(125)^{\frac{2}{3}}}{(32)^{\frac{2}{5}}}$$

$$= \frac{(5 \times 5 \times 5)^{\frac{2}{3}}}{(2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}}}$$

$$= \frac{\left(5^3\right)^{\frac{2}{3}}}{\left(2^5\right)^{\frac{2}{5}}}$$

$$= \frac{5^2}{2^2}$$

$$= \frac{25}{4}$$

$$= 6\frac{1}{4}$$

(iv)

$$\left[1 - \left\{1 - (1 - n)^{-1}\right\}^{-1}\right]^{-1} = \frac{1}{\left[1 - \left\{1 - (1 - n)^{-1}\right\}^{-1}\right]^{+1}}$$

$$= \frac{1}{1 - \frac{1}{1 - (1-n)^{-1}}}$$

$$= \frac{1}{1 - \frac{1}{1 - \frac{1}{(1-n)}}}$$

$$= \frac{1}{1 - \frac{1}{\frac{1(1-n) - 1}{(1-n)}}}$$

$$= \frac{1}{1 - \frac{1}{\frac{1 - n - 1}{(1-n)}}}$$

$$= \frac{1}{1 - \frac{1}{\frac{-n}{(1-n)}}}$$

$$= \frac{1}{1 - \frac{(1-n)}{-n}}$$

$$= \frac{1}{1 + \frac{(1-n)}{n}}$$

$$= \frac{1}{\frac{n + (1-n)}{n}}$$

$$= \frac{1}{\frac{n + 1 - n}{n}}$$

$$= \frac{n}{1}$$

$$= n$$

Solution 5:

$2160 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2^4 \times 3^3 \times 5^1 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2^a \times 3^b \times 5^c = 2^4 \times 3^3 \times 5^1$

Comparing powers of 2,3 and 5 on the both sides of equation, we have

a=4;b=3 and c=1

Hence value of $3^a \times 2^{-b} \times 5^{-c} = 3^4 \times 2^{-3} \times 5^{-1}$

$$= 3 \times 3 \times 3 \times 3 \times \frac{1}{2^3} \times \frac{1}{5}$$

$$= 81 \times \frac{1}{2 \times 2 \times 2} \times \frac{1}{5}$$

$$= 81 \times \frac{1}{8} \times \frac{1}{5}$$

$$= \frac{81}{40}$$

$$= 2\frac{1}{40}$$

Solution 6:

$1960 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2 \times 2 \times 2 \times 5 \times 7 \times 7 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2^3 \times 5^1 \times 7^2 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2^a \times 5^b \times 7^c = 2^3 \times 5^1 \times 7^2$

Comparing powers of 2,5 and 7 on the both sides of equation, we have

a=3;b=1 and c=2

Hence value of $2^{-a} \times 7^b \times 5^{-c} = 2^{-3} \times 7^1 \times 5^{-2}$

$$= \frac{1}{2^3} \times 7 \times \frac{1}{5^2}$$

$$= \frac{1}{8} \times 7 \times \frac{1}{5 \times 5}$$

$$= \frac{7}{200}$$

Solution 7:

(i)

$$\frac{8^{3a}\times 2^{5}\times 2^{2a}}{4\times 2^{11a}\times 2^{-2a}}=\frac{\left(2^{3}\right)^{3a}\times 2^{5}\times 2^{2a}}{2^{2}\times 2^{11a}\times 2^{-2a}}$$

$$=\frac{2^{3\times 3a}\times 2^{5}\times 2^{2a}}{2^{2}\times 2^{11a}\times 2^{-2a}}$$

$$=\frac{2^{9a}\times 2^{5}\times 2^{2a}}{2^{2}\times 2^{11a}\times 2^{-2a}}$$

$$=2^{9a+5+2a-2-11a+2a}$$

$$=2^{2a+3}$$

(ii)

$$\frac{3\times 27^{n+1}+9\times 3^{3n-1}}{8\times 3^{3n}-5\times 27^{n}}=\frac{3\times (3\times 3\times 3)^{n+1}+3\times 3\times 3^{3n-1}}{2\times 2\times 2\times 3^{3n}-5\times (3\times 3\times 3)^{n}}$$

$$=\frac{3\times \left(3^{3}\right)^{n+1}+3^{2}\times 3^{3n-1}}{2^{3}\times 3^{3n}-5\times \left(3^{3}\right)^{n}}$$

$$=\frac{3\times 3^{3n+3}+3^{3n+1}}{2^{3}\times \left(3^{3}\right)^{n}-5\times \left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n+3+1}+3^{3n+1}}{2^{3}\times \left(3^{3}\right)^{n}-5\times \left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n+4}+3^{3n+1}}{2^{3}\times \left(3^{3}\right)^{n}-5\times \left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n}\times 3^{4}+3^{3n}\times 3^{1}}{2^{3}\times \left(3^{3}\right)^{n}-5\times \left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n}\left(3^{4}+3^{1}\right)}{\left(3^{3}\right)^{n}(8-5)}$$

$$=\frac{3^{3n}\left(3^{4}+3^{1}\right)}{3^{3n}\times 3}$$

$$=\frac{3\times 3\times 3\times 3+3}{3}$$

$$=\frac{81+3}{3}$$

$$=\frac{84}{3}$$

$$=28$$

Solution 8:

$$\left(\frac{a^m}{a^{-n}}\right)^{m-n} \times \left(\frac{a^n}{a^{-\ell}}\right)^{n-\ell} \times \left(\frac{a^\ell}{a^{-m}}\right)^{\ell-m}$$

$$= \left(a^m \times a^n\right)^{m-n} \times \left(a^n \times a^\ell\right)^{n-\ell} \times \left(a^\ell \times a^m\right)^{\ell-m}$$

$$= \left(a^{m+n}\right)^{m-n} \times \left(a^{n+\ell}\right)^{n-\ell} \times \left(a^{\ell+m}\right)^{\ell-m}$$

$$= a^{m^2-n^2} \times a^{n^2-\ell^2} \times a^{\ell^2-m^2}$$

$$= a^{m^2-n^2+n^2-\ell^2+\ell^2-m^2}$$

$$= a^0$$

$$= 1$$

Solution 9:

$$a = x^{m+n}.x^l$$

$$b = x^{n+l}.x^m$$

$$c = x^{l+m}.x^n$$

LHS

$$a^{m-n}.b^{n-l}.c^{l-m}$$

$$= \left(x^{m+n}.x^l\right)^{m-n}.\left(x^{n+l}.x^m\right)^{n-l}.\left(x^{l+m}.x^n\right)^{l-m} \text{ [Substituting a,b,c in LHS]}$$

$$= x^{(m+n)(m-n)}.x^{l(m-n)}.x^{(n+l)(n-l)}.x^{m(n-l)}.x^{(l+m)(l-m)}.x^{n(l-m)}$$

$$= x^{m^2-n^2+ml-nl+n^2-l^2+mn-nl+l^2-m^2+nl-mn}$$

$$= x^0$$

$$= 1 = \text{RHS}$$

Solution 10:

(i)

$$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$$

$$= \left(x^{a-b}\right)^{a^2+ab+b^2} \times \left(x^{b-c}\right)^{b^2+bc+c^2} \times \left(x^{c-a}\right)^{c^2+ca+a^2}$$

$$= x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3}$$

$$= x^{a^3-b^3+b^3-c^3+c^3-a^3}$$

$$= x^0$$

$$= 1$$

(ii)

$$\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$$

$$= \left(x^{a+b}\right)^{a^2-ab+b^2} \times \left(x^{b+c}\right)^{b^2-bc+c^2} \times \left(x^{c+a}\right)^{c^2-ca+a^2}$$

$$= x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3}$$

$$= x^{a^3+b^3+b^3+c^3+c^3+a^3}$$

$$= x^{\left(a^3+b^3+b^3+c^3+c^3+a^3\right)}$$

$$= x^{2\left(a^3+b^3+c^3\right)}$$

Exercise 7(B)

Solution 1:

(i)

$2^{2x+1} = 8$

$\Rightarrow 2^{2x+1} = 2^3$

We know that if bases are equal, the powers are equal

$\Rightarrow 2x+1=3$

$\Rightarrow 2x=3-1$

$\Rightarrow 2x = 2$

$\Rightarrow x = \frac{2}{2}$

$\Rightarrow x = 1$

(ii)

$2^{5x-1} = 4 \times 2^{3x+1}$

$\Rightarrow 2^{5x-1} = 2^2 \times 2^{3x+1}$

$\Rightarrow 2^{5x-1} = 2^{3x+1+2}$

$\Rightarrow 2^{5x-1} = 2^{3x+3}$

We know that if bases are equal, the powers are equal

$\Rightarrow 5x-1=3x+3$

$\Rightarrow 5x-3x=3+1$

$\Rightarrow 2x = 4$

$\Rightarrow x = \frac{4}{2}$

$\Rightarrow x = 2$

(iii)

$3^{4x+1} = 27^{(x+1)}$

$\Rightarrow 3^{4x+1} = \left(3^3\right)^{x+1}$

$\Rightarrow 3^{4x+1} = 3^{3x+3}$

We know that if bases are equal, the powers are equal

$\Rightarrow 4x+1=3x+3$

$\Rightarrow 4x-3x=3-1$

$\Rightarrow x = 2$

(iv)

$49^{x+4} = 7^2\left(343\right)^{(x+1)}$

$\Rightarrow \left(7 \times 7\right)^{x+4} = 7^2\left(7 \times 7 \times 7\right)^{(x+1)}$

$\Rightarrow \left(7^2\right)^{x+4} = 7^2\left(7^3\right)^{(x+1)}$

$\Rightarrow 7^{2x+8} = 7^2 \times 7^{3x+3}$

$\Rightarrow 7^{2x+8} = 7^{3x+3+2}$

$\Rightarrow 7^{2x+8} = 7^{3x+5}$

We know that if bases are equal, the powers are equal

$\Rightarrow 2x+8=3x+5$

$\Rightarrow 3x-2x=8-5$

$\Rightarrow x = 3$

Solution 2:

(i)

$$4^{2x} = \frac{1}{32}$$

$$\Rightarrow (2 \times 2)^{2x} = \frac{1}{2 \times 2 \times 2 \times 2 \times 2}$$

$$\Rightarrow \left(2^2\right)^{2x} = \frac{1}{2^5}$$

$$\Rightarrow 2^{2x2x} = 2^{-5}$$

$$\Rightarrow 2^{4x} = 2^{-5}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow 4x = -5$$

$$\Rightarrow x = \frac{-5}{4}$$

(ii)

$$\sqrt{2^{x+3}} = 16$$

$$\left(2^{x+3}\right)^{\frac{1}{2}} = 2 \times 2 \times 2 \times 2$$

$$\Rightarrow 2^{\frac{x+3}{2}} = 2^4$$

We know that if bases are equal, the powers are equal

$$\Rightarrow \frac{x+3}{2} = 4$$

$$\Rightarrow x + 3 = 8$$

$$\Rightarrow x = 8 - 3$$

$$\Rightarrow x = 5$$

(iii)

$$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$$

$$\Rightarrow \left[\left(\frac{3}{5}\right)^{\frac{1}{2}}\right]^{x+1} = \frac{5 \times 5 \times 5}{3 \times 3 \times 3}$$

$$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \left(\frac{5}{3}\right)^{3}$$

$$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \left(\frac{3}{5}\right)^{-3}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow \frac{x+1}{2} = -3$$

$$\Rightarrow x + 1 = -6$$

$$\Rightarrow x = -6 - 1$$

$$\Rightarrow x = -7$$

(iv)

$$\left(\sqrt[3]{\frac{2}{3}}\right)^{x-1} = \frac{27}{8}$$

$$\left[\left(\frac{2}{3}\right)^{\frac{1}{3}}\right]^{x-1} = \frac{3^3}{2^3}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{3}} = \left(\frac{3}{2}\right)^{3}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{3}} = \left(\frac{2}{3}\right)^{-3}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow \frac{x-1}{3} = -3$$

$$\Rightarrow x - 1 = -9$$

$$\Rightarrow x = -9 + 1$$

$$\Rightarrow x = -8$$

Solution 3:

(i)

$4^{x-2} - 2^{x+1} = 0$

$\Rightarrow 4^{x-2} = 2^{x+1}$

$\Rightarrow (2^2)^{x-2} = 2^{x+1}$

$\Rightarrow 2^{2x-4} = 2^{x+1}$

We know that if bases are equal, the powers are equal

$\Rightarrow 2x - 4 = x + 1$

$\Rightarrow 2x - x = 4 + 1$

$\Rightarrow x = 5$

(ii)

$3^{x^2} : 3^x = 9 : 1$

$\frac{3^{x^2}}{3^x} = \frac{9}{1}$

$\Rightarrow 3^{x^2} = 9 \times 3^x$

$\Rightarrow 3^{x^2} = 3^2 \times 3^x$

$\Rightarrow 3^{x^2} = 3^{x+2}$

We know that if bases are equal, the powers are equal

$\Rightarrow x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow x(x - 2) + 1(x - 2) = 0$

$\Rightarrow (x + 1)(x - 2) = 0$

$\Rightarrow x + 1 = 0$ or $x - 2 = 0$

$\Rightarrow x = -1$ or $x = 2$

Solution 4:

(i)

$$8 \times 2^{2x} + 4 \times 2^{x+1} = 1 + 2^x$$

$$\Rightarrow 8 \times \left(2^x\right)^2 + 4 \times 2^x \times 2^1 = 1 + 2^x$$

$$\Rightarrow 8 \times \left(2^x\right)^2 + 4 \times \left(2^x\right) \times 2^1 - 1 - 2^x = 0$$

$$\Rightarrow 8 \times \left(2^x\right)^2 + \left(2^x\right) \times (8 - 1) - 1 = 0$$

$$\Rightarrow 8 \times \left(2^x\right)^2 + 7\left(2^x\right) - 1 = 0$$

$$\Rightarrow 8y^2 + 7y - 1 = 0 \quad [y = 2^x]$$

$$\Rightarrow 8y^2 + 8y - y - 1 = 0$$

$$\Rightarrow 8y(y + 1) - 1(y + 1) = 0$$

$$\Rightarrow (8y - 1)(y + 1) = 0$$

$$\Rightarrow 8y = 1 \text{ or } y = -1$$

$$\Rightarrow y = \frac{1}{8} \text{ or } y = -1$$

$$\Rightarrow 2^x = \frac{1}{8} \text{ or } 2^x = -1$$

$$\Rightarrow 2^x = \frac{1}{2^3} \text{ or } 2^x = -1$$

$$\Rightarrow 2^x = 2^{-3} \text{ or } 2^x = -1$$

$$\Rightarrow x = -3$$

$[\because 2^x = -1 \text{ is not possible}]$

(ii)

$2^{2x}+2^{x+2}-4\times 2^3=0$

$\Rightarrow\left(2^x\right)^2+2^x\cdot 2^2-4\times 2\times 2\times 2=0$

$\Rightarrow\left(2^x\right)^2+2^x\cdot 2^2-32=0$

$\Rightarrow y^2+4y-32=0 \quad [y=2^x]$

$\Rightarrow y^2+8y-4y-32=0$

$\Rightarrow y(y+8)-4(y+8)=0$

$\Rightarrow (y+8)(y-4)=0$

$\Rightarrow y+8=0$ or $y-4=0$

$\Rightarrow y=-8$ or $y=4$

$\Rightarrow 2^x=-8$ or $2^x=4$

$\Rightarrow 2^x=2^2 \quad [\because 2^x=-8$ is not possible$]$

$\Rightarrow x=2$

(iii)

$\left(\sqrt{3}\right)^{x-3}=\left(\sqrt[4]{3}\right)^{x+1}$

$\Rightarrow\left(3^{\frac{1}{2}}\right)^{x-3}=\left(3^{\frac{1}{4}}\right)^{x+1}$

$\Rightarrow 3^{\frac{x-3}{2}}=3^{\frac{x+1}{4}}$

$\Rightarrow \frac{x-3}{2}=\frac{x+1}{4}$

$\Rightarrow 4(x-3)=2(x+1)$

$\Rightarrow 4x-12=2x+2$

$\Rightarrow 4x-2x=12+2$

$\Rightarrow 2x=14$

$\Rightarrow x=\frac{14}{2}$

$\Rightarrow x=7$

Solution 5:

$$4^{2m}=\left(\sqrt[3]{16}\right)^{-\frac{6}{n}}=\left(\sqrt{8}\right)^2$$

$$\Rightarrow 4^{2m}=\left(\sqrt{8}\right)^2 \ldots.(1)$$

and

$$\left(\sqrt[3]{16}\right)^{-\frac{6}{n}}=\left(\sqrt{8}\right)^2 \ldots.(2)$$

From (1)

$$4^{2m}=\left(\sqrt{8}\right)^2$$

$$\Rightarrow \left(2^2\right)^{2m}=\left(\sqrt{2^3}\right)^2$$

$$\Rightarrow 2^{4m}=\left[\left(2^3\right)^{\frac{1}{2}}\right]^2$$

$$\Rightarrow 2^{4m}=\left[2^{3\times\frac{1}{2}}\right]^2$$

$$\Rightarrow 2^{4m}=2^{3\times\frac{1}{2}\times 2}$$

$$\Rightarrow 2^{4m}=2^3$$

$$\Rightarrow 4m=3$$

$$\Rightarrow m=\frac{3}{4}$$

From (2), we have

$$\left(\sqrt[3]{16}\right)^{-\frac{6}{n}}=\left(\sqrt{8}\right)^2$$

$$\Rightarrow \left(\sqrt[3]{2\times2\times2\times2}\right)^{-\frac{6}{n}}=\left(\sqrt{2\times2\times2}\right)^2$$

$$\Rightarrow \left(\sqrt[3]{2^4}\right)^{-\frac{6}{n}}=\left(\sqrt{2^3}\right)^2$$

$$\Rightarrow \left[\left(2^4\right)^{\frac{1}{3}}\right]^{-\frac{6}{n}}=\left[\left(2^3\right)^{\frac{1}{2}}\right]^2$$

$$\Rightarrow \left[2^{\frac{4}{3}}\right]^{-\frac{6}{n}}=\left[2^{\frac{3}{2}}\right]^2$$

$$\Rightarrow 2^{\frac{4}{3}\times\left(-\frac{6}{n}\right)}=2^{\frac{3}{2}\times 2}$$

$$\Rightarrow 2^{\left(-\frac{8}{n}\right)}=2^3$$

$$\Rightarrow -\frac{8}{n}=3$$

$$\Rightarrow n=\frac{-8}{3} \qquad \text{Thus } m=\frac{3}{4}\ \ n=\frac{-8}{3}$$

Solution 6:

Consider the equation

$$\left(\sqrt{32}\right)^{x} \div 2^{y+1} = 1$$

$$\Rightarrow \left(\sqrt{2\times 2\times 2\times 2\times 2}\right)^{x} \div 2^{y+1} = 1$$

$$\Rightarrow \left(\sqrt{2^5}\right)^{x} \div 2^{y+1} = 1$$

$$\Rightarrow \left[\left(2^5\right)^{\frac{1}{2}}\right]^{x} \div 2^{y+1} = x^0$$

$$\Rightarrow 2^{\frac{5x}{2}} \div 2^{y+1} = x^0$$

$$\Rightarrow \frac{5x}{2} - (y+1) = 0$$

$$\Rightarrow 5x - 2(y+1) = 0$$

$$\Rightarrow 5x - 2y - 2 = 0.....(1)$$

Now consider the other equation

$$8^{y} - 16^{4-\frac{x}{2}} = 0$$

$$\Rightarrow \left(2^3\right)^{y} - \left(2^4\right)^{4-\frac{x}{2}} = 0$$

$$\Rightarrow 2^{3y} - 2^{4\left(4-\frac{x}{2}\right)} = 0$$

$$\Rightarrow 2^{3y} = 2^{4\left(4-\frac{x}{2}\right)}$$

$$\Rightarrow 3y = 4\left(4-\frac{x}{2}\right)$$

$$\Rightarrow 3y = 16 - 2x$$

$$\Rightarrow 2x + 3y = 16.....(2)$$

Thus we have two equations,

$5x - 2y = 2$ .....(1)

$2x + 3y = 16$.....(2)

Multiplying (1) by 3 and (2) by 2, we have

$15x - 6y = 6$....(3)

$4x + 6y = 32$....(4)

Adding (3) and (4), we have

$19x=38$

$\Rightarrow x=2$

Substituting the value of x in equation (1), we have,

$5(2) - 2y = 2$

$\Rightarrow 10 - 2y = 2$

$\Rightarrow 2y = 10 - 2$

$\Rightarrow 2y = 8$

$\Rightarrow y = \frac{8}{2}$

$\Rightarrow y = 4$

Thus the values of x and y are:

$x=2$ and $y=4$

Solution 7:

(i)

$$\text{L.H.S.} = \left(\frac{x^a}{x^b}\right)^{a+b-c} \times \left(\frac{x^b}{x^c}\right)^{b+c-a} \times \left(\frac{x^c}{x^a}\right)^{c+a-b}$$

$$= \left(x^{a-b}\right)^{(a+b-c)} \times \left(x^{b-c}\right)^{(b+c-a)} \times \left(x^{c-a}\right)^{(c+a-b)}$$

$$= x^{(a-b)(a+b-c)} \times x^{(b-c)(b+c-a)} \times x^{(c-a)(c+a-b)}$$

$$= x^{a^2+ab-ac-ab-b^2+bc} \times x^{b^2+bc-ab-cb-c^2+ac} \times x^{c^2+ac-bc-ac-a^2+ab}$$

$$= x^{a^2-ac-b^2+bc+b^2-ab-c^2+ac+c^2-bc-a^2+ab}$$

$$= x^0$$

$$= 1$$

$$= \text{R.H.S}$$

(ii)

We need to prove that

$$\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = 1$$

$$\text{L.H.S.} = x^{a(b-c)-b(a-c)} \div \frac{x^{bc}}{x^{ac}}$$

$$\Rightarrow \quad = x^{ab-ac-ab+bc} \div x^{bc-ac}$$

$$\Rightarrow \quad = x^{ab-ac-ab+bc-(bc-ac)}$$

$$\Rightarrow \quad = x^{ab-ac-ab+bc-bc+ac}$$

$$\Rightarrow \quad = x^0$$

$$\Rightarrow \quad = 1$$

$$\Rightarrow \quad = \text{R.H.S}$$

Solution 8:

We are given that

$a^x = b, b^y = c$ and $c^z = a$

Consider the equation

$a^x = b$

$\Rightarrow a^{xyz} = b^{yz}$ [raising to the power yz on both sides]

$\Rightarrow a^{xyz} = \left(b^y\right)^z$

$\Rightarrow a^{xyz} = (c)^z$ $[\because b^y = c]$

$\Rightarrow a^{xyz} = c^z$

$\Rightarrow a^{xyz} = a$ $[\because c^z = a]$

$\Rightarrow a^{xyz} = a^1$

$\Rightarrow xyz = 1$

Solution 9:

Let $a^x = b^y = c^z = k$

$\therefore a = k^{\frac{1}{x}}; b = k^{\frac{1}{y}}; c = k^{\frac{1}{z}}$

Also, we have $b^2 = ac$

$$\therefore \left(k^{\frac{1}{y}}\right)^2 = \left(k^{\frac{1}{x}}\right) \times \left(k^{\frac{1}{z}}\right)$$

$$\Rightarrow k^{\frac{2}{y}} = k^{\frac{1}{x}+\frac{1}{z}}$$

$$\Rightarrow k^{\frac{2}{y}} = k^{\frac{z+x}{xz}}$$

Comparing the powers we have

$$\frac{2}{y} = \frac{z+x}{xz}$$

$$\Rightarrow y = \frac{2xz}{z+x}$$

Solution 10:

Let $5^{-p} = 4^{-q} = 20^{r} = k$

$5^{-p} = k \Rightarrow 5 = k^{-\frac{1}{p}} [\because a^{p} = b^{q} \Rightarrow a = b^{\frac{q}{p}}]$

$4^{-q} = k \Rightarrow 4 = k^{-\frac{1}{q}} [\because a^{p} = b^{q} \Rightarrow a = b^{\frac{q}{p}}]$

$20^{r} = k \Rightarrow 20 = k^{\frac{1}{r}} [\because a^{p} = b^{q} \Rightarrow a = b^{\frac{q}{p}}]$

$5 \times 4 = 20$

$\Rightarrow k^{-\frac{1}{p}} \times k^{-\frac{1}{q}} = k^{\frac{1}{r}}$

$\Rightarrow k^{-\frac{1}{p}-\frac{1}{q}} = k^{\frac{1}{r}}$

$\Rightarrow k^{0} = k^{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}$

$\Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 0$ [If bases are equal, powers are also equal]

Solution 11:

$(m+n)^{-1}(m^{-1}+n^{-1}) = m^{x}n^{y}$

$\Rightarrow \frac{1}{(m+n)} \times \left(\frac{1}{m} + \frac{1}{n}\right) = m^{x}n^{y}$

$\Rightarrow \frac{1}{(m+n)} \times \left(\frac{m+n}{mn}\right) = m^{x}n^{y}$

$\Rightarrow \frac{1}{mn} = m^{x}n^{y}$

$\Rightarrow m^{-1}n^{-1} = m^{x}n^{y}$

Comparing the coefficients of x and y, we get

$x = -1$ and $y = -1$

LHS,

$x + y + 2 = (-1) + (-1) + 2 = 0 = RHS$

Solution 12:

$5^{x+1} = 25^{x-2}$

$\Rightarrow 5^{x+1} = \left(5^2\right)^{x-2}$

$\Rightarrow 5^{x+1} = 5^{2x-4}$ [If bases are equal, powers are also equal]

$\Rightarrow x + 1 = 2x - 4$

$\Rightarrow 2x - x = 4 + 1$

$\Rightarrow x = 5$

$\therefore 3^{x-3} \times 2^{3-x} = 3^{5-3} \times 2^{3-5} = 3^2 \times 2^{-2} = 9 \times \frac{1}{4} = \frac{9}{4}$

Solution 13:

$4^{x+3} = 112 + 8 \times 4^x$

$\Rightarrow 4^x \times 4^3 = 112 + 8 \times 4^x$

$\Rightarrow 64 \times 4^x = 112 + 8 \times 4^x$

Let $4^x = y$

$64y = 112 + 8y$

$\Rightarrow 56y = 112$

$\Rightarrow y = 2$

Substituting we get,

$4^x = 2$

$\Rightarrow 2^{2x} = 2$

$\Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

$(18x)^{3x} = \left(\frac{18}{2}\right)^{3 \times \frac{1}{2}} = 9^{3 \times \frac{1}{2}} = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$

Solution 14(i):

(i)

$$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^{-x}$$

$$\Rightarrow \left(2^2\right)^{x-1} \times \left(\frac{1}{2}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^{-x}$$

$$\Rightarrow 2^{2x-2} \times 2^{-(3-2x)} = \left(2^{-3}\right)^{-x}$$

$$\Rightarrow 2^{2x-2-3+2x} = 2^{3x}$$

$$\Rightarrow 2^{4x-5} = 2^{3x}$$

$$\Rightarrow 4x - 5 = 3x$$

$$\Rightarrow 4x - 3x = 5$$

$$\Rightarrow x = 5$$

Solution 14(ii):

$$a^{2(3x+5)} \times a^{4x} = a^{8x+12}$$

$$\Rightarrow a^{6x+10+4x} = a^{8x+12}$$

$\Rightarrow 10x + 10 = 8x + 12$ [If bases are the same, powers are also same]

$$\Rightarrow 2x = 2$$

$$\Rightarrow x = 1$$

Solution 14(iii):

$$(81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{-\frac{2}{5}} + x\left(\frac{1}{2}\right)^{-1} . 2^{0} = 27$$

$$\Rightarrow 3^{4 \times \frac{3}{4}} - \left(2^{-5}\right)^{-\frac{2}{5}} + x\,(2) = 27$$

$$\Rightarrow 3^{3} - 2^{2} + 2x = 27$$

$$\Rightarrow 2x + 27 - 4 = 27$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

Solution 14(iv):

$2^{3x} \times 2^3 = 2^{3x} \times 2 + 48$

$\Rightarrow 8 \times 2^{3x} = 2^{3x} \times 2 + 48$

$\Rightarrow 2^{3x} (8 - 2) = 48$

$\Rightarrow 2^{3x} \times 6 = 48$

$\Rightarrow 2^{3x} = 8$

$\Rightarrow 2^{3x} = 2^3$

$\Rightarrow 3x = 3$

$\Rightarrow x = 1$

Solution 14(v):

$3 \times 2^x + 3 - 2^x \times 2^2 + 5 = 0$

$\Rightarrow 2^x (3 - 4) + 8 = 0$

$\Rightarrow -2^x = -8$

$\Rightarrow 2^x = 8$

$x=3$

Exercise 7(C)

Solution 1:

(i) $9^{\frac{5}{2}} - 3 \times 8^{0} - \left(\frac{1}{81}\right)^{-\frac{1}{2}}$

$= \left(3^{2}\right)^{\frac{5}{2}} - 3 \times 1 - \left(\frac{1}{3^{4}}\right)^{-\frac{1}{2}}$

$= 3^{2 \times \frac{5}{2}} - 3 - 3^{-4 \times \left(-\frac{1}{2}\right)}$

$= 3^{5} - 3 - 3^{2}$

$= 243 - 3 - 9$

$= 231$

(ii) $(64)^{\frac{2}{3}} - \sqrt[3]{125} - \frac{1}{2^{-5}} + (27)^{-\frac{2}{3}} \times \left(\frac{25}{9}\right)^{-\frac{1}{2}}$

$= \left(4^{3}\right)^{\frac{2}{3}} - \sqrt[3]{5^{3}} - 2^{5} + \left(3^{3}\right)^{-\frac{2}{3}} \times \left(\frac{5^{2}}{3^{2}}\right)^{-\frac{1}{2}}$

$= 4^{2} - 5 - 2^{5} + 3^{-2} \times \left(\frac{5}{3}\right)^{2 \times \left(-\frac{1}{2}\right)}$

$= 16 - 5 - 32 + \frac{1}{3^{2}} \times \left(\frac{5}{3}\right)^{-1}$

$= -21 + \frac{1}{9} \times \frac{3}{5}$

$= -21 + \frac{1}{15}$

$= \frac{-315 + 1}{15}$

$= \frac{-314}{15}$

$= -20\frac{14}{15}$

$$(iii)\ \left[\left(-\frac{2}{3}\right)^{-2}\right]^{3} \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$$

$$= \left[\left(-\frac{3}{2}\right)^{2}\right]^{3} \times (3)^{4} \times \frac{1}{3} \times \frac{1}{3 \times 2}$$

$$= \left(-\frac{3}{2}\right)^{6} \times (3)^{2} \times \frac{1}{2}$$

$$= \frac{3^{6+2}}{2^{6+1}}$$

$$= \frac{3^{8}}{2^{7}}$$

Solution 2:

$$\frac{3 \times 9^{n+1} - 9 \times 3^{2n}}{3 \times 3^{2n+3} - 9^{n+1}}$$

$$= \frac{3 \times \left(3^{2}\right)^{n+1} - 3^{2} \times 3^{2n}}{3 \times 3^{2n+3} - \left(3^{2}\right)^{n+1}}$$

$$= \frac{3^{1+2n+2} - 3^{2+2n}}{3^{1+2n+3} - 3^{2n+2}}$$

$$= \frac{3^{3+2n} - 3^{2+2n}}{3^{4+2n} - 3^{2n+2}}$$

$$= \frac{3^{2n}\left(3^{3} - 3^{2}\right)}{3^{2n}\left(3^{4} - 3^{2}\right)}$$

$$= \frac{27 - 9}{81 - 9}$$

$$= \frac{18}{72}$$

$$= \frac{1}{4}$$

Solution 3:

$$3^{x-1} \times 5^{2y-3} = 225$$

$$\Rightarrow 3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$$

$$\Rightarrow x - 1 = 2 \text{ and } 2y - 3 = 2$$

$$\Rightarrow x = 3 \text{ and } 2y = 5$$

$$\Rightarrow x = 3 \text{ and } y = \frac{5}{2}$$

$$\Rightarrow x = 3 \text{ and } y = 2\frac{1}{2}$$

Solution 4:

$$\left(\frac{a^{-1}b^2}{a^2b^{-4}}\right)^7 \div \left(\frac{a^3b^{-5}}{a^{-2}b^3}\right)^{-5} = a^x \cdot b^y$$

$$\Rightarrow \left(\frac{b^6}{a^3}\right)^7 \div \left(\frac{a^5}{b^8}\right)^{-5} = a^x \cdot b^y$$

$$\Rightarrow \left(\frac{b^6}{a^3}\right)^7 \div \left(\frac{b^8}{a^5}\right)^{5} = a^x \cdot b^y$$

$$\Rightarrow \frac{b^{42}}{a^{21}} \div \frac{b^{40}}{a^{25}} = a^x \cdot b^y$$

$$\Rightarrow \frac{b^{42}}{a^{21}} \times \frac{a^{25}}{b^{40}} = a^x \cdot b^y$$

$$\Rightarrow b^2 \times a^4 = a^x \times b^y$$

$$\Rightarrow x = 4 \text{ and } y = 2$$

$$\Rightarrow x + y = 4 + 2 = 6$$

Solution 5:

$$3^{x+1} = 9^{x-3}$$

$$\Rightarrow 3^x \times 3 = \left(3^2\right)^{x-3}$$

$$\Rightarrow 3^x \times 3 = 3^{2x-6}$$

$$\Rightarrow 3^x \times 3 = \frac{3^{2x}}{3^6}$$

$$\Rightarrow 3^6 \times 3 = \frac{3^{2x}}{3^x}$$

$$\Rightarrow 3^7 = 3^x$$

$$\Rightarrow x = 7$$

$$\Rightarrow 2^{1+x} = 2^{1+7} = 2^8 = 256$$

Solution 6:

$2^x = 4^y = 8^z$

$\Rightarrow 2^x = 2^{2y} = 2^{3z}$

$\Rightarrow x = 2y = 3z$

$\Rightarrow y = \frac{x}{2}$ and $z = \frac{x}{3}$

Now, $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z} = 4$

$\Rightarrow \frac{1}{2x} + \frac{1}{\frac{4x}{2}} + \frac{1}{\frac{8x}{3}} = 4$

$\Rightarrow \frac{1}{2x} + \frac{2}{4x} + \frac{3}{8x} = 4$

$\Rightarrow \frac{1}{2x} + \frac{1}{2x} + \frac{3}{8x} = 4$

$\Rightarrow \frac{4+4+3}{8x} = 4$

$\Rightarrow \frac{11}{8x} = 4$

$\Rightarrow x = \frac{11}{32}$

Solution 7:

$\frac{9^n \cdot 3^2 \cdot 3^n - (27)^n}{(3^m \cdot 2)^3} = 3^{-3}$

$\Rightarrow \frac{3^{2n} \cdot 3^2 \cdot 3^n - 3^{3n}}{3^{3m} \cdot 2^3} = \frac{1}{3^3}$

$\Rightarrow \frac{3^{3n} \cdot 3^2 - 3^{3n}}{3^{3m} \cdot 2^3} = \frac{1}{3^3}$

$\Rightarrow \frac{3^{3n}(3^2 - 1)}{3^{3m} \times 8} = \frac{1}{3^3}$

$\Rightarrow \frac{3^{3n} \times 8}{3^{3m} \times 8} = \frac{1}{3^3}$

$\Rightarrow \frac{1}{3^{3(m-n)}} = \frac{1}{3^{3\times 1}}$

$\Rightarrow m - n = 1$ (proved)

Solution 8:

$$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$$

$$\Rightarrow (13)^{\sqrt{x}} = 256 - 81 - 6$$

$$\Rightarrow (13)^{\sqrt{x}} = 169$$

$$\Rightarrow (13)^{\sqrt{x}} = 13^2$$

$$\Rightarrow \sqrt{x} = 2$$

$$\Rightarrow x = 4$$

Solution 9:

$$3^{4x} = (81)^{-1} \text{ and } (10)^{\frac{1}{y}} = 0.0001$$

$$\Rightarrow 3^{4x} = \left(3^4\right)^{-1} \text{ and } (10)^{\frac{1}{y}} = \frac{1}{10000}$$

$$\Rightarrow 3^{4x} = 3^{-4} \text{ and } (10)^{\frac{1}{y}} = \frac{1}{10^4}$$

$$\Rightarrow 4x = -4 \text{ and } (10)^{\frac{1}{y}} = 10^{-4}$$

$$\Rightarrow x = -1 \text{ and } \frac{1}{y} = -4$$

$$\Rightarrow x = -1 \text{ and } y = -\frac{1}{4}$$

$$\therefore 2^{-x} \times 16^{y} = 2^{-(-1)} \times 16^{-\frac{1}{4}}$$

$$= 2 \times 2^{4\times\left(-\frac{1}{4}\right)}$$

$$= 2 \times 2^{-1}$$

$$= 2^{1-1}$$

$$= 2^0$$

$$= 1$$

Solution 10:

$$3(2^x + 1) - 2^{x+2} + 5 = 0$$
$$\Rightarrow 3 \times 2^x + 3 - 2^x \times 2^2 + 5 = 0$$
$$\Rightarrow 2^x (3 - 2^2) + 8 = 0$$
$$\Rightarrow 2^x (3 - 4) = -8$$
$$\Rightarrow 2^x \times (-1) = -8$$
$$\Rightarrow 2^x = 8$$
$$\Rightarrow 2^x = 2^3$$
$$\Rightarrow x = 3$$

Solution 11:

$$\left(a^m\right)^n = a^m \cdot a^n$$
$$\Rightarrow a^{mn} = a^{m+n}$$
$$\Rightarrow mn = m + n \quad \ldots.(1)$$

Now,

$$m(n-1) - (n-1)$$
$$= mn - m - n + 1$$
$$= m + n - m - n + 1 \quad \ldots.[\text{From } (1)]$$
$$= 1$$

Solution 12:

$$m = \sqrt[3]{15} \text{ and } n = \sqrt[3]{14}$$
$$\Rightarrow m^3 = 15 \text{ and } n^3 = 14$$
$$\therefore m - n - \frac{1}{m^2 + mn + n^2} = \frac{(m^3 + m^2n + mn^2) - (m^2n + mn^2 + n^3) - 1}{m^2 + mn + n^2}$$
$$= \frac{m^3 + m^2n + mn^2 - m^2n - mn^2 - n^3 - 1}{m^2 + mn + n^2}$$
$$= \frac{m^3 - n^3 - 1}{m^2 + mn + n^2}$$
$$= \frac{15 - 14 - 1}{m^2 + mn + n^2}$$
$$= \frac{1 - 1}{m^2 + mn + n^2}$$
$$= 0$$